## Mathematics

# Fairfield Public Schools

## ALGEBRA 31

## ALGEBRA 31

### Critical Areas of Focus

Building on their work with linear, quadratic, and exponential functions from Algebra, students in Algebra 31 will extend their repertoire of functions to include polynomial, rational, and radical functions. Students work closely with the expressions that define the functions, and continue to expand and hone their abilities to model situations and to solve equations, including solving quadratic equations over the set of complex numbers and solving exponential equations using the properties of logarithms. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations. The critical areas for this course are as follows:

1. In this area, students synthesize and generalize what they have learned about a variety of function families. They extend their work with exponential functions to include solving exponential equations with logarithms. They explore the effects of transformations on graphs of diverse functions, including functions arising in an application, in order to abstract the general principle that transformations on a graph always have the same effect regardless of the type of the underlying function. They identify appropriate types of functions to model a situation, they adjust parameters to improve the model, and they compare models by analyzing appropriateness of fit and making judgments about the domain over which a model is a good fit. The description of modeling as "*the process of choosing and using mathematics and statistics to analyze empirical situations, to understand them better, and to make decisions*" is at the heart of this area. The narrative discussion and diagram of the modeling cycle should be considered when knowledge of functions, statistics, and geometry is applied in a modeling context.

2. This area develops the structural similarities between the system of polynomials and the system of integers. Students draw on analogies between polynomial arithmetic and base-ten computation, focusing on properties of operations, particularly the distributive property. Students connect multiplication of polynomials with multiplication of multi-digit integers, and division of polynomials with long division of integers. Students identify zeros of polynomials, including complex zeros of quadratic polynomials, and make connections between zeros of polynomials and solutions of polynomial equations. The unit culminates with the fundamental theorem of algebra. A central theme is that the arithmetic of rational expressions is governed by the same rules as the arithmetic of rational numbers.

3. Building on their previous work with functions in Algebra, and on their work with trigonometric ratios and circles in Geometry, students now use the coordinate plane to extend trigonometry to model periodic phenomena.

4. In this area, students see how the visual displays and summary statistics they learned in earlier grades relate to different types of data and to probability distributions. They identify different ways of collecting data—including sample surveys, experiments, and simulations—and the role that randomness and careful design play in the conclusions that can be drawn.

| Pacing Guide | | | |
|---|---|---|---|
| 1st Marking Period | 2nd Marking Period | 3rd Marking Period | 4th Marking Period |
| September     October     November | December     January     February | March     April | May     June |

| Unit 1 | Unit 2 | Unit 3 | Unit 4 | Unit 5 | Unit 6 |
|---|---|---|---|---|---|
| **Rational and Polynomial Expressions** | **Exponential Functions** | **Probability** | **Statistics Random Processes** | **Sequences and Series** | **Introduction to Trigonometry** |
| 8 weeks | 8 weeks | 4 weeks | 4 weeks | 4 weeks | 6 weeks |

| Course Overview | | |
|---|---|---|
| **Central Understandings** | **Essential Questions** | **Assessments** |
| Insights learned from exploring generalizations through the essential questions. (Students should understand that…) <br>● Patterns and functional relationships can be represented and analyzed using a variety of strategies, tools, and technologies. <br>● Quantitative relationships can be expressed numerically in multiple ways in order to make connections and simplify calculations using a variety of strategies, tools, and technologies. <br>● Shapes and structures can be analyzed, visualized, measured, and transformed using a variety of strategies, tools, and technologies. <br>● Data can be analyzed to make informed decisions using a variety of strategies, tools, and technologies. | ● How do patterns and functions help us describe data and physical phenomena and solve a variety of problems? <br>● How are quantitative relationships represented by numbers? <br>● How do geometric relationships and measurements help us to solve problems and make sense of our world? <br>● How can collecting, organizing and displaying data help us analyze information and make reasonable and informed decision? | ● Formative Assessments <br>● Summative Assessments |

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| Content Outline | Standards |
|---|---|
|    I.   Unit 1 – Rational and Polynomial Expressions<br>  II.   Unit 2 – Exponential Functions<br> III.   Unit 3 – Probability<br> IV.   Unit 4 – Statistics & Random Processes<br>  V.   Unit 5 – Sequences & Series<br> VI.   Unit 6  - Introduction to Trigonometry | Connecticut Common Core State Standards are met in the following areas:<br><br>● ***The Number Systems***<br>● ***Expressions and Equations***<br>● ***Number and Quantity***<br>● ***Algebra***<br>● ***Functions***<br>● ***Statistics and Probability*** |

| Algebra 31 Standards for Mathematical Practice | |
|---|---|
| The K-12 Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. This page gives examples of what the practice standards look like at the specified grade level.  Students are expected to: | |
| ***Standards*** | ***Explanations and Examples*** |
| **1. Make sense of problems and persevere in solving them**. | In Algebra, students solve problems involving equations and discuss how they solved them. Students solve real world problems through the application of algebraic and geometric concepts. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?", and "Can I solve the problem in a different way?" |
| **2. Reason abstractly and quantitatively.** | This practice standard refers to one of the hallmarks of algebraic reasoning, the process of de-contextualization and contextualization. Much of elementary algebra involves creating abstract algebraic models of problems and then transforming the models via algebraic calculations (A-SSE, A-APR, F-IF) to reveal properties of the problems. |
| **3. Construct viable arguments and critique the reasoning of others.** | In Algebra, students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays (i.e. box plots, dot plots, histograms, etc.). They further refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. They pose questions like "How did you get that?", "Why is that true?" "Does that always work?" They explain their thinking to others and respond to others' thinking. |
| **4. Model with mathematics.** | Indeed, other mathematical practices in Algebra 2 might be seen as contributing specific elements of these two. The intent of the following set is not to decompose the above mathematical practices into component parts but rather to show how the mathematical practices work together. |
| **5. Use appropriate tools strategically.** | Students consider available tools such as spreadsheets, a function modeling language, graphing tools and many other technologies so they can strategically gain understanding of the ideas expressed by individual content standards and to model with mathematics. |
| **6. Attend to precision.** | In Algebra, the habit of using precise language is not only a mechanism for effective communication but also a tool for understanding and solving problems. Describing an idea precisely helps students understand the idea in new ways. |
| **7. Look for and make use of structure.** | In Algebra, students should look for various structural patterns that can help them understand a problem.  For example, writing $49x^2 + 35x + 6$ as $(7x)^2 + 5(7x) + 6$ is  a practice many teachers refer to as "chunking," highlights the structural similarity between this expression and $z^2 + 5z + 6$, leading to a factorization of the original: $((7x) + 3) ((7x) + 2)$. |
| **8. Look for and express regularity in repeated reasoning**. | Creating equations or functions to model situations is harder for many students than working with the resulting expressions. An effective way to help students develop the skill of describing general relationships is to work through several specific examples and then express what they are doing with algebraic symbolism. For example, when comparing two different text messaging plans, many students who can compute the cost for a given number of minutes have a hard time writing general formulas that express the cost of each plan for *any* number of minutes. Constructing these formulas can be facilitated by methodically calculating the cost for several different input values and then expressing the steps in the calculation, first in words and then in algebraic symbols. Once such expressions are obtained, students can find the break-even point for the two plans, graph the total cost against the number of messages sent and make a complete analysis of the two plans. |

| **Unit 1 Rational and Polynomial Expressions, 8 weeks  top** |
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This unit focuses on performing operations on rational and polynomial expressions and simplifying them. In order to understand the rational expressions, students review the real number system.

| **Big Ideas**<br>The central organizing ideas and underlying structures of mathematics | **Essential Questions** |
|:---:|:---:|
| <ul><li>The complex numbers are numbers that can be written in the form $a + bi$, where $i$ denotes a square root of -1.</li><li>The fundamental theorem of algebra states that every non-constant single-variable polynomial with complex coefficients has at least one complex root.</li></ul> | <ul><li>How can we extend from real numbers to complex numbers?</li><li>How can we solve equations with complex and real roots?</li><li>How can we extend known rules of functions to analyze the structure of rational and polynomial expressions?</li></ul> |
| **Common Core State Standards** ||

*NUMBER AND QUANTITY*
*The Real Number System N-RN*
**Use properties of rational and irrational numbers.**

    **N-RN 3**
    Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.


*The Complex Number System N-CN*
**Perform arithmetic operations with complex numbers.**

    **N-CN 1**
    Know there is a complex number $i$ such that $i^2 = -1$, and every complex number has the form $a + bi$ with $a$ and $b$ real.
    **N-CN 2**
    Use the relation $i^2 = -1$, and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
    **N.CN.8**
    (+) Extend polynomial identities to the complex numbers. *For example, rewrite $x^2 + 4$ as $(x + 2i)(x – 2i)$.*
    **N.CN.9**
    (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

**Use complex numbers in polynomial identities and equations.**
    **N-CN 7**
    Solve quadratic equations with real coefficients that have complex solutions.

*ALGEBRA*

*Seeing Structure in Expressions A-SSE*
**Interpret the structure of expressions.**

**A-SSE 1**
Interpret expressions that represent a quantity in terms of its context.*

**A-SSE 1a**
Interpret parts of an expression, such as terms, factors, and coefficients.

**A-SSE 1b**
Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.*

**A-SSE 2**
Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.*

*Arithmetic with Polynomials and Rational Expressions A-APR*
**Perform arithmetic operations on polynomials**

**A-APR 1**
Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

**Understand the relationship between zeros and factors of polynomials.**

**A-APR 2**
Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number $a$, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.

**A-APR 3**
Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

*Arithmetic with Polynomials and Rational Expressions A-APR*
**Use polynomial identities to solve problems.**

**A-APR 4**
Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.*

**Rewrite rational expressions.**

**A-APR 6**
Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.

**Represent and solve equations and inequalities graphically.**

**A-REI 11**

Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.*

*(+)* denotes a standard in Algebra 31, but not in Algebra 32

* *Denotes a modeling standard in the CCSS*

## Unit 2 – Exponential Functions,  8 weeks  <u>top</u>

This unit builds on the students' experience in Algebra 1 where the students were first exposed to exponential functions in one of their initial units.  The students expand their understanding that an exponential function has a common multiplier, thus affecting the initial value in either a growth or decay model. Additionally, the students in Algebra 2 will expand their understanding of the laws of exponents by introducing rational powers. In conclusion of the unit, the students will be introduced to the inverse function.  This next logical step allows students to use inverse functions to solve equations (logarithms; logarithmic functions as inverses of exponential functions; the laws of exponents and logarithm).

| **Big Ideas**<br>The central organizing ideas and underlying structures of mathematics | **Essential Questions** |
|---|---|
| <ul><li>Functions can be represented in various ways, including through algebraic means (e.g., equations), graphs, word descriptions, and table.</li><li>Exponential functions are characterized by a rate of change that is proportional to the value of the function.  It is property of exponential functions that whenever the input is increased by 1 unit, the output is multiplied by a constant factor.</li><li>Exponential functions connect multiplication through addition through the equation $a^{m+n} = \left(a^m\right)\left(a^n\right)$.</li><li>Under appropriate conditions, functions have inverses.  For example, logarithmic functions are inverse functions of exponential functions and the square root function is the inverse of quadratic function.</li></ul> | <ul><li>How do exponential functions help us create models to describe data and physical phenomenon and solve problems</li><li>How do we apply properties of exponents to whole number and rational exponents?</li><li>How do we apply properties of exponents to whole number and rational exponents and their relationship to logarithms?</li><li>How do we use technology to work with graphs and tables to solve problems and make sense of our world?</li><li>How do we extend known rules of functions to exponential functions (transformations and operations)?</li><li>How do we build functions to model sequences and series?</li></ul> |

### Common Core State Standards

*NUMBER AND QUANTITY*
*The Real Number System N-RN*
**Extend the properties of exponents to rational exponents.**
> **N-RN 1**
> Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.*
> **N-RN 2**
> Rewrite expressions involving radicals and rational exponents using the properties of exponents

*ALGEBRA*
*Seeing Structure in Expressions A-SSE*
**Write expressions in equivalent forms to solve problems.**

    **A-SSE 3c**. Use the properties of exponents to transform expressions for exponential functions. *For example the expression* $1.15^t$ *can be rewritten as* $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ *to reveal the approximate equivalent* monthly interest rate if the annual rate is 15%.

*Creating Equations A-CED*
**Create equations that describe numbers or relationships.**

    **A-CED 1**

    Create equations and inequalities in one variable and use them to solve problems. Include equations arising from ~~linear and quadratic functions, and~~ simple rational and exponential functions.

*Reasoning with Equations and Inequalities A-REI*
**Understand solving equations as a process of reasoning and explain the reasoning.**

    **A-REI 2**

    Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
**Represent and solve equations and inequalities graphically.**

    **A-REI 11**

    Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.*

*FUNCTIONS*
*Interpreting Functions F-IF*
**Understand the concept of a function and use function notation.**

    **F-IF 3**

    Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1*

**Analyze functions using different representations.**

    **F-IF 7e**

    Graph exponential and logarithmic functions, showing intercepts and end behavior.

    **F-IF 8**

    Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

        **F-IF 8b**

        Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as

        $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay.

*Building functions F-BF*
**Build a function that models a relationship between two quantities.**
> **F-BF 1**
> Write a function that describes a relationship between two quantities.
>> **F-BF 1b**
>> Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.
>
> **F-BF 2**
> Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
>
> F-BF 3
> Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of $k$ (both positive and negative); find the value of $k$ given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

**Build new functions from existing functions.**
> **F-BF 4**
> Find inverse functions.
>> **F-BF 4a**
>> Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. *For example, f(x)* $=2x^3$ *or f(x)* $= \frac{x+1}{x-1}$ *for x ≠ 1.*

*Linear, Quadratic, and Exponential Models\* F-LE*
**Construct and compare linear, quadratic, and exponential models and solve problems**.
> **F-LE 1**
> Distinguish between situations that can be modeled with linear functions and with exponential functions.
>> **F-LE 1a**
>> Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
>> **F-LE 1b**
>> Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
>> **F-LE 1c**
>> Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
>
> **F-LE 2**
> Construct ~~linear and~~ exponential functions … given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
>
> **F-LE 3**
> Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

**F-LE 4**
For exponential models, express as a logarithm the solution $ab^{ct} = d$ where $a$, $c$, d and $t$ are numbers and the base $b$ is 2, 10, or $e$; evaluate the logarithm using technology. Interpret expressions for functions in terms of the situation they model.

**F-LE 5**
Interpret the parameters in a linear or an exponential function in terms of a context.

This unit builds upon knowledge of probability first learned in seventh grade that includes uses of random sampling to draw inferences, compares inferences between populations, investigates chance processes, and develops, uses and evaluates probability models. The unit introduces foundational properties and rules of probabilities, starting with events as subsets of sample space and introducing independence and conditional probability. The unit includes treatment of rules of probability to determine likelihood of compound events and introduces counting principals to calculate probability.

| **Big Ideas**<br>The central organizing ideas and underlying structures of mathematics | **Essential Questions** |
|---|---|
| ● A conditional probability is the probability that an event will occur, when another event is known to occur or to have occurred.<br>● Two events are independent if the occurrence of one does not affect the probability of the other.<br>● The use of probability can provide a justification to help make a decision. | ● How are counting principals applied to probability calculations?<br>● How do we utilize various probability rules? (ex. Addition Rule, conditional, independence…)<br>● How can we utilize diagrams, tables and technology to organize data and solve problems? |

**Common Core State Standards**

*STATISTICS AND PROBABILITY*
*Conditional Probability and the Rules of Probability S-CP*
**Understand independence and conditional probability and use them to interpret data.**

**S-CP 1**
Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").

**S-CP 2**
Understand that two events $A$ and $B$ are independent if the probability of $A$ and $B$ occurring together is the product of their probabilities, and use this characterization to determine if they are independent.

**S-CP 3**
Understand the conditional probability of $A$ given $B$ as $P(A$ and $B)/P(B)$, and interpret independence of $A$ and $B$ as saying that the conditional probability of $A$ given $B$ is the same as the probability of $A$, and the conditional probability of $B$ given $A$ is the same as the probability of $B$.

**S-CP 4**
Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*

**S-CP 5**
Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*

**Use the rules of probability to compute probabilities of compound events in a uniform probability model.**

**S-CP 6**
(+) Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A* and interpret the answer in terms of the model.
**S-CP 7**
(+) Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.

*Using Probability to Make Decisions S-MD*
**Use probability to evaluate outcomes of decisions.**
**S.MD.6**
(+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).
**S.MD.7**
(+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

*(+)* denotes a standard in Algebra 31, but not in Algebra 32
* denotes a modeling standard in the CCSS

## Unit 4 – Statistics (Random Processes), 4 weeks  <span style="color:blue">top</span>

This unit builds on the prior high school units of probability and statistics and involves significant use of modeling. The treatment begins with understanding the random processes that underlie statistical experiments. The unit connects processes of inference from statistics with tools and techniques from probability. This includes taking random samples to make judgments about a population and determining whether a model is consistent from the empirical data retrieved. Surveys, experiments, simulations and observational studies are used to collect and study data. Students make inferences and justify conclusions from analyzing the margin of error, examining the standard deviation of distributions, comparing difference between populations, and calculating whether a difference is significant. The unit culminates with a project in which a situation is modeled, analyzed, and a concluding report is generated.

| Big Ideas<br>The central organizing ideas and underlying structures of mathematics | Essential Questions |
|---|---|
| <ul><li>Statistical models extend mathematical models by describing variability around the structure.</li><li>Observational studies, including surveys, provide information about the characteristics of a population or sample, whereas controlled experiments provide information about treatment effects.</li><li>Random assignment in an experiment permits drawing causal conclusions about treatment effects and quantifying the uncertainty associated with these conclusions.</li><li>Random selection tends to produce samples that are representative of the population, permitting generalization from the sample to the larger population and also allowing the uncertainty in estimates to be quantified.</li><li>Random selection and random assignment are different things, and the type and scope of conclusions that can be drawn from data depend on the role of random selection and random assignment in the study.</li><li>Estimating an estimator involves considering bias, precision, and the sampling method.</li></ul> | <ul><li>How are normal distributions used to solve problems?</li><li>How do we analyze data sets to form hypotheses and make predictions?</li><li>How do we collect, organize and display data using appropriate statistical and graphical methods?</li></ul> |

### Common Core State Standards

*STATISTICS AND PROBABILTIY*
*Making Inferences and Justifying Conclusions S-IC*
**Understand and evaluate random processes underlying statistical experiments.**
> **S-IC 1**
> Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
> **S-IC 2**
> Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?

*Making Inferences and Justifying Conclusions S-IC*

**Make inferences and justify conclusions from sample surveys, experiments, and observational studies.**

> **S-IC 3**
> Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.
> **S-IC 4**
> Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.
> **S-IC 5**
> Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.
> **S-IC 6**
> Evaluate reports based on data.

*Interpreting Categorical and Quantitative Data S-ID*
**Summarize, represent, and interpret data on a single count or measurement variable.**

> **S-ID 4**
> Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

## Unit 5 – Sequences and Series,  4 weeks  <span style="color:blue">top</span>

In this unit, students will explore sequences and series.  They will define explicit rules that generate number sequences whose terms have a common difference or a common ratio, and they use summation notation to represent and find the sum of terms of a series. Students will also use rules for the sum of arithmetic series, finite geometric series, and infinite geometric series, and infinite geometric series.

| Big Ideas<br>The central organizing ideas and underlying structures of mathematics | Essential Questions |
|---|---|
| <ul><li>Functions can be represented in various ways, including through algebraic means (e.g., equations), graphs, word descriptions, and table.</li><li>Geometric sequences can be thought of as exponential functions whose domains are the positive integers.</li><li>Arithmetic sequences can be thought of as linear functions where the domains are the positive integers.</li></ul> | <ul><li>How can you write an expression for sums such as 5 + 10 + 15 + 50?</li><li>How can you tell that a sequence is arithmetic or geometric?</li><li>When does a geometric series have a sum, and when does it not have a sum?</li><li>How do you write a recursive rule for a arithmetic sequence and a geometric sequence?</li><li>How do you translate between recursive and explicit formulas for arithmetic and geometric sequences?</li></ul> |

### Common Core State Standards

*ALGEBRA*
*Seeing Structure in Expressions A-SSE*
**Interpret the structure of expressions.**

    **A-SSE 1**
    Interpret expressions that represent a quantity in terms of its context.*

        **A-SSE 1a**
        Interpret parts of an expression, such as terms, factors, and coefficients.

        **A-SSE 1b**
        Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.*

**Write expressions in equivalent forms to solve problems.**

    **A-SSE 4**
    Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments.*

*Building functions F-BF*
**Build a function that models a relationship between two quantities.**

    **F-BF 1**
    Write a function that describes a relationship between two quantities.

    **F-BF 2**

Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

## Unit 6: Introduction to Trigonometry, 6 weeks   top

Trigonometric ratios can be thought of as functions of the angles. With the help of the unit circle, the angles do not need to be between 0 and 90 degrees. By extending the domains to all real numbers, these trigonometric functions are used to model circular and periodic motions.

| Big Ideas The central organizing ideas and underlying structures of mathematics | Essential Questions |
|---|---|
| <ul><li>Trigonometric functions are natural and fundamental examples of periodic functions.</li><li>For angles between 0° and 90°, the trigonometric functions can be defined as ratios of side lengths in right triangles.  These functions are well defined because the ratios of side lengths are equivalent in similar triangles.</li><li>For general angles, the sine and cosine functions can be viewed as the $y$- and $x$-coordinates of points on circles or as the projection of circular motion onto the $y$- and $x$-axes.</li><li>Functions can be represented in various ways, including through algebraic means (e.g., equations), graphs, word descriptions, and table.</li></ul> | <ul><li>How do we extend known rules of functions to trigonometric functions (transformations and operations)?</li><li>How do we graph by hand and using technology?</li><li>How do we model periodic data to solve problems?</li></ul> |

### Common Core State Standards

*FUNCTIONS*
*Interpreting Functions F-IF*
**Analyze functions using different representations.**
> **F-IF 7**
> Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
>> **F-IF 7e**
>> Graph … trigonometric functions, showing period, midline, and amplitude.

*Trigonometric Functions F-TF*
**Extend the domain of trigonometric functions using the unit circle.**
> **F-TF 1**
> Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
> **F-TF 2**
> Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

**Model periodic phenomena with trigonometric functions.**

    **F-TF 5**

    Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.*

**Prove and apply trigonometric identities.**

    **F-TF 8**

    Prove the Pythagorean identity $sin^2(\theta) + cos^2(\theta) = 1$ and use it to find $sin(\theta)$, $cos(\theta)$, or $tan(\theta)$ given $sin(\theta)$, $cos(\theta)$, or $tan(\theta)$ and the quadrant of the angle.

*\* denotes a modeling standard in the CCSS*