Geometry

Course Information

Grade(s):	9-12
Discipline/Course:	Mathematics / Geometry
Course Title:	Geometry
Prerequisite(s):	Successful completion of one of the following: Algebra I Algebra I H
Course Description: *Program of Studies*	The purpose of the Geometry course is to formalize and extend students' geometric experiences in the middle grades. Students explore more complex geometric situations and deepen their explanation of geometric relationships, moving towards formal mathematical arguments. The Mathematical Practice Standards apply throughout each course and, together with the Common Core State Standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations. The critical areas of focus for this course are on geometric figures and theorems, transformations, similarity and congruence, analysis of right triangles and trigonometry, two and three dimensional objects, coordinate Geometry and circles. The course also includes a unit of statistics and probability to support students' understanding of concepts essential to quantitative literacy.
Course Essential Questions:	● How are patterns, algebra and geometry related? ● What connections exist between transformations and dilations and congruence and similarity? ● How do you develop a convincing argument? ● How do you analyze the relationship between geometric figures? ● How can quantitative literacy support our understanding of thinking logically and solving problems?
Course Enduring Understandings:	● Mathematics can be used to solve problems outside of the mathematics classroom. ● Mathematics is built on reason and justification. ● Reasoning allows us to make conjectures and to prove conjectures.

	● Classifying helps us to build networks of mathematical ideas. ● Precise language helps us express mathematical ideas and receive them.
Duration: **Credit:**	Full Year 1.0 Credit(s)
Course Materials/Resources:	Pre-AP Geometry with Statistics Framework *Illustrative Mathematics Geometry*- McGraw Hill (2020)
FPS Course Academic Expectation(s):	Exploring and Understanding Using Communication Tools
Year at a Glance (Units):	Unit 1: Tools and Techniques of Geometric Measurement (~ 10-12 weeks) Unit 2: Measurement in Congruent and Similar Figures (~10-12 weeks) Unit 3: Measurement in Two and Three Dimensions (~ 4-6 weeks) Unit 4: Measurement in Data (~ 5-7 weeks)

Unit Number and Title:	Unit 1: Tools and Techniques of Geometric Measurement
Duration:	~ 10-12 weeks
Unit Overview:	This unit introduces students to the basic objects of geometry and the tools used to explore these objects throughout the remainder of the course. The basic objects students investigate in this unit include lines, rays, segments, and angles. These figures serve as the building blocks of more complex objects that students explore in later units. Students continue to expand their understanding of measurement by developing techniques for quantifying and comparing the attributes of geometric objects. The tools they use to analyze objects may include straightedges, compasses, rulers, protractors, dynamic geometry software, the coordinate plane, and right triangles. In addition, students use an informal understanding of transformations throughout the unit to justify whether two basic objects are congruent. They formalize transformations and define congruence and similarity through transformations. This course contains an introduction to right triangle trigonometry, which integrates the tools and techniques of the unit into an investigation of new ways to express the relationship between angle measures and side lengths. Throughout the course, specific learning objectives require students to prove geometric concepts. Students' proofs can be organized in a variety of formats, such as two-column tables or paragraphs. The format of a student's proof is not as important as their ability to justify a mathematical claim or provide a counterexample disproving one. They should develop an understanding that a mathematical proof establishes the truth of a statement by combining previously developed truths into a logically consistent argument.
Learning Goals	
Standard(s):	**GEOMETRY** **Congruence (CO)** **Experiment with transformations in the plane.** G.CO.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line

segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

Understand congruence in terms of rigid motions.

G.CO.6

Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

Prove geometric theorems.

G.CO.9

Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints*.

G.CO.10

Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point*.

Make geometric constructions.

G.CO.12

Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line*.

Expressing Geometric Properties with Equations (GPE)

Use coordinates to prove simple geometric theorems algebraically.

	G.GPE.5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point). G.GPE.6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio. **Similarity, Right Triangles, and Trigonometry (SRT)** **Prove theorems involving similarity.** G.SRT.4 Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity. **Define trigonometric ratios and solve problems involving right triangles.** G.SRT.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. G.SRT.7 Explain and use the relationship between the sine and cosine of complementary angles. G.SRT.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. **Modeling with Geometry (MG)** **Apply geometric concepts in modeling situations.** G.MG.1 Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). G.MG.3

	Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). **NUMBER AND QUANTITY** **Quantities (Q)** **Reason quantitatively and use units to solve problems.** N.Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. N.Q.2 Define appropriate quantities for the purpose of descriptive modeling. N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
Essential Question(s):	● How do geometric relationships and measurements help us to solve problems and make sense of our world? ● How do mathematical ideas interconnect and build on one another to produce a coherent whole? ● How does geometry model the physical world? ● How can the language of geometry be used to communicate mathematical ideas coherently and precisely? ● How does the language of geometry provide immediate experience with the physical world? ● How does the geometric principle of congruence apply to the real world? ● How do parallel lines, transversals, and related angles model the physical world? ● How do triangles, their sides, angles, and special segments model the physical world?
Enduring Understanding(s):	● A formal mathematical argument establishes new truths by logically combining previously known facts. ● Measuring features of geometric figures is the process of assigning numeric values to attributes

	of the figures, which allows the attributes to be compared. ● Pairs of lines in a plane that never intersect or that intersect at right angles have special geometric and algebraic properties. ● Right triangles are simple geometric shapes in which we can relate the measures of acute angles to ratios of their side lengths.
Learning Goal(s): *Students will know and will be able to use their learning to:* (Content/ Skills)	**Content: (**Students will know/understand…) **Measurement in Geometry:** ● The structure of a conditional statement is in the form of an *If-Then* statement, because IF a hypothesis is met, THEN the conclusion is performed. ● The converse of a conditional statement is created when the hypothesis and conclusion are reversed. ● A biconditional statement is a relation between two propositions that is true only when both propositions are simultaneously true or false. ● For any two distinct points in a plane, there is only one line that contains them. ● A point is an exact location in space. It has no dimensions. ● A plane is an endless flat surface with no width. ● A line is straight, has no width, extends infinitely in two directions, and contains infinitely many points. A line can be named by a single lowercase letter, or it can be named by any two distinct points on that line. ● A ray is a portion of a line that has a single endpoint and extends infinitely in one direction. A ray can be named by its endpoint and any other point on the ray, with its endpoint listed first. ● A line segment is a portion of a line between and including two endpoints. A line segment can be named by its two endpoints. ● An intersection is the point or points that geometric figures have in common. ● An angle is a geometric figure formed when two lines, line segments, or rays share an endpoint. The point common to both lines, line segments, or rays is called the vertex of the angle. ● An angle can be named by its vertex. An angle can also be named using its vertex and the names of a non-vertex point that lies on each of its sides. For such angle names, the point that indicates the vertex is the second of the three points.

	• The length of a line segment is the distance between its endpoints and is measured using a specific unit of measure. • An angle can be measured by determining the amount of rotation one ray would make about the vertex of the angle to coincide with the other ray. The amount of rotation is measured as a fraction of the rotation needed to rotate a full circle. • An angle can be measured with reference to a circle whose center is the vertex of the angle by determining the fraction of the circular arc between the intersection points of the rays and the circle. The length of the circular arc is measured as a fraction of the circle's circumference. • An angle can be measured in units of radians, equaling the arc length spanned by the angle when its vertex coincides with the center of a unit circle. • Two line segments are congruent if and only if one segment can be translated, rotated, or reflected to coincide with the other segment without changing the length of either line segment. • Two line segments are congruent if and only if they have the equal lengths. • Two angles are congruent if and only if one segment can be translated, rotated, or reflected to coincide with the other angle without changing the measure of either angle. • Two angles are congruent if and only if they have the equal measures. • Make geometric constructions with a variety of tools and methods. • The distance between two points in the plane is the length of the line segment connecting the points. • Use proof and logical reasoning to justify Geometric relationships and concepts. • The distance between two points in the coordinate plane can be determined by applying the Pythagorean Theorem to a right triangle whose hypotenuse is a line segment formed by the two points and whose sides are parallel to each axis. • Given the line segment $\overline{AC}$ and a point B, that lines on the segment between points A and C, the measure of segment $\overline{AC}$ is the sum of the measures of segments $\overline{AB}$ and $\overline{BC}$. • Given ∠AOC and ray $\vec{OB}$ that lies between $\vec{OA}$ and $\vec{OC}$, the measure of ∠AOC is equal to the sum of the measures of ∠AOB and ∠BOC. • Two angles are called complementary if the sum of their measures is 90°. Two angles are complementary if they form a right angle when adjacent.

	● Two angles are called supplementary if the sum of their measures is 180°. Two angles are supplementary if they form a straight angle when adjacent. ● The midpoint of a line segment is the point located on the line segment equidistant from the endpoints. ● In the coordinate plane, the x- and y- coordinates of the midpoint of a line segment are the arithmetic means of the corresponding coordinates of the endpoints. ● A bisector of an angle is a line, ray, or line segment that contains the vertex of the angle and divides the angle into two congruent adjacent angles. ● Points that lie on the angle bisector are equidistant from the sides of the angle. **<u>Parallel and Perpendicular Lines:</u>** ● The relationship between the slopes of parallel lines can be justified by comparing their slope triangles using translation. ● The relationship between the slopes of perpendicular lines can be justified by comparing their slope triangles using rotation by 90°. ● Two distinct lines, rays, or line segments in the coordinate plane are parallel if and only if they have the same slope or are both vertical. ● A transversal is a line that intersects a set of lines. Two lines, rays, or line segments intersected by a transversal will be parallel if and only if the same-side interior angles formed by the lines and the transversal are supplementary. ● Two lines intersected by a transversal will be parallel if and only if the corresponding angles, alternate interior angles, or alternate exterior angles formed by the lines and the transversals of lines are congruent. ● Given a line and a point not on the given line, there is exactly one line through the point that will be parallel to the given line. ● Two parallel lines, rays, or line segments in the coordinate plane will have equal slopes and contain no common points. ● The sum of the interior angles of a triangle in a plane is 180°. ● Isosceles Triangles are triangles with at least two congruent sides. Their base angles are congruent to each other.

	- Equilateral Triangles are triangles with three congruent sides. All their angles are congruent to each other. - A line, ray, or line segment is perpendicular to another line, ray, or line segment if and and only if they form right angles at the point where the two figures intersect. - A line, ray, or line segment is perpendicular to another line, ray, or line segment in the coordinate plane if and only if the two figures intersect and their slopes are opposite reciprocals of each other, or one is vertical and the other is horizontal. - The perpendicular bisector of a line segment that intersects the line segment at its midpoint and forms four right angles with the line segment. - The perpendicular bisector of a line segment is determined by identifying two points in a plane that are equidistant from the endpoints of the line segment and constructing a line, ray, or line segment through those two points. - Every point that lies on the perpendicular bisector of a line segment is equidistant from the endpoints of the line segment. - A horizontal line, ray, or line segment in the coordinate plane is perpendicular to a vertical line, ray, or line segment if they intersect. - Two perpendicular lines, rays, or line segments in the coordinate plane will intersect and have slopes that are opposite reciprocals of each other, or one will be vertical and the other will be horizontal. - Applying the perpendicular bisector construction to a point on a line, ray, or line segment is sufficient to construct a line, ray, or line segment perpendicular to the given line, ray or line segment. **Measurement in Right Triangles:** - Two right triangles are similar if and only if one triangle can be translated, reflected, and/or rotated so it coincides with the other after dilating one triangle by a scale factor. - Two right triangles are similar if and only if their corresponding angles have equal measures. - Two right triangles are similar if and only if their corresponding side lengths are in proportion. - The coordinates of a point along a line segment in the coordinate plane that divides the line segment into a given ratio can be determined using similar triangles.

- An altitude drawn from the right angle of a right triangle to the hypotenuse creates similar right triangles.
- When an altitude is constructed from the right angle to the hypotenuse of a right triangle, the proportions of the sides lengths of the similar right triangles formed can be used to prove the Pythagorean Theorem.
- The 30-60-90 Triangle and 45-45-90 Triangle have specific ratios of their sides.
- The sine of the measure of $\angle A$ is the ratio of the length of the side opposite the angle and the length of the hypotenuse.
- The cosine of the measure of $\angle A$ is the ratio of the length of the side adjacent the angle and the length of the hypotenuse.
- The tangent of the measure of $\angle A$ is the ratio of the length of the side opposite the angle and the length of the adjacent to the angle.
- Trigonometric ratios are functions whose input is an acute angle measure and whose output is a ratio of two side lengths in a right triangle.
- The ratio of the lengths of two sides of a right triangle will equal the ratio of the lengths of the corresponding sides of a similar right triangle. Therefore, the ratios of the sides depend only on the angle measure.
- For acute angles in a right triangle, the angle measure and the ratio of the lengths of any two specific sides have a one-to-one correspondence.
- Given a ratio of any two side lengths in a right triangle, it is possible to determine the acute angle measures of the right triangle.
- Contextual scenarios involve nonvertical and non-horizontal segments or the distance between two points that do not lie on a vertical or horizontal line can be modeled by right triangles.
- Trigonometric ratios can be used to solve problems or model scenarios involving angles of elevation and depression.

Skills: (Students will be able to…)

- Form and analyze conditional, converse, and biconditional statements.
- Describe and correctly label a line, ray, line segment, and angle.
- Measure a line segment, and an angle.

	● Prove whether two or more line segments are congruent, or whether two or more angles are congruent. ● Construct a congruent copy of a line segment or an angle. ● Calculate the distance between two points. ● Solve problems involving segment lengths and/or angle measures. ● Solve problems involving a segment bisector or an angle bisector. ● Justify the relationship between the slopes of parallel or perpendicular lines in the coordinate plane using transformations. ● Solve problems involving two or more parallel lines, rays, or line segments. ● Solve problems involving the triangle sum theorem. ● Solve problems involving two or more perpendicular lines, rays, or line segments. ● Construct the perpendicular bisector of a line segment. ● Construct a line, ray, or line segment perpendicular to another line, ray or line segment. ● Prove whether two right triangles are similar using informal similarity transformations. ● Determine the coordinates of a point on a line segment. ● Prove the Pythagorean Theorem using similar right triangles. ● Apply 30-60-90 Triangle Ratios. ● Apply 45-45-90 Triangle Ratios. ● Associate the measures of an acute angle, $\angle A$, in a right triangle to ratios of side lengths. ● Explain why a trigonometric ratio depends only on an angle measure of a right triangle and not on the side lengths. ● Determine an acute angle measure in a right triangle, given a ratio of its side lengths, using an understanding of inverses. ● Solve problems using the relationship between sine and cosine of complementary angles. ● Model contextual scenarios using right triangles.

Unit Number and Title:	Unit 2: Measurement in Congruent and Similar Figures
Duration:	~ 10-12 weeks
Unit Overview:	Informal transformations compare two objects to see if they are congruent. We slide (translate), twist (rotate), and flip (reflect) objects to see if one can lay exactly on the other without bending, stretching, or breaking either object. When they match, we say the objects are congruent. If they do not match, but they have the same shape and the same scaled measurements, we say the objects are similar. Transformations in geometry give us language to describe these slides, twists, flips, and scaling precisely and systematically. This unit formalizes the concept of congruence and similarity of planar objects by identifying the essential components of rigid motion and similarity transformations. Students are expected to become proficient with transformations that involve coordinates as well as with transformations that do not involve coordinates. Throughout the course, transformations are presented as functions. This connection further develops students' understanding of functions and connects to the statistics and geometry units of the course. It also creates a bridge between Algebra 1 and Algebra 2 since the concept of function permeates and links nearly all aspects of high school mathematics. Students develop further insights into congruence and similarity by exploring which transformations affect angle measures and distances between pairs of points and which do not. Students apply their understandings of transformations, congruence, and similarity to solve problems involving polygons and circles. Throughout the course, specific learning objectives require students to prove geometric concepts. Students' proofs can be organized in a variety of formats, such as two-column tables, or paragraphs. The format of a student's proof is not as important as their ability to justify a mathematical claim or provide a counterexample disproving one. They should develop an understanding that a mathematical proof establishes the truth of a statement by combining previously developed truths into a logically consistent argument.

Learning Goals	
Standard(s):	**GEOMETRY** **Congruence (CO)** **Experiment with transformations in the plane.** G.CO.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). G.CO.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself. G.CO.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. G.CO.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another. **Understand congruence in terms of rigid motions.** G.CO.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. G.CO.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

G.CO.8
Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

Prove geometric theorems.

G.CO.11
Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.

Circles (C)

Understand and apply theorems about circles.

G.C.1
Prove that all circles are similar.

G.C.2
Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.

G.C.3
Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.

G.C.4 (+)
Construct a tangent line from a point outside a given circle to the circle.

Find arc lengths and areas of sectors of circles.

G.C.5
Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

	Expressing Geometric Properties with Equations (GPE) **Translate between the geometric description and the equation for a conic section.** G.GPE 1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. G.GPE.4 Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point $(1, \sqrt{3})$ lies on the circle centered at the origin and containing the point (0, 2). **Similarity, Right Triangles, and Trigonometry (SRT)** **Understand similarity in terms of similarity transformations.** G.SRT.1 Verify experimentally the properties of dilations given by a center and a scale factor: a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor. G.SRT.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. G.SRT.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar. **Prove theorems involving similarity.** G.SRT.4

	Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity. G.SRT.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. **Define trigonometric ratios and solve problems involving right triangles.** G.SRT.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. G.SRT.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. **Modeling with Geometry (MG)** **Apply geometric concepts in modeling situations.** G.MG.1 Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). G.MG.3 Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). **NUMBER AND QUANTITY** **Quantities (Q)** **Reason quantitatively and use units to solve problems.** N.Q.1 Use units as a way to understand problems and to guide the solution of multi-step

	problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. N.Q.2 Define appropriate quantities for the purpose of descriptive modeling. N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
Essential Question(s):	● How is similarity used to measure indirectly and explore comparable objects? ● How are appropriate techniques, tools, and formulas used in geometry to determine measurements? ● How is trigonometry used to understand the functional and aesthetic uses of right triangles? ● How do triangles, their sides, angles, and special segments model the physical world? ● How do transformations provide a way of studying figures? ● How does the geometric principle of congruence in triangles apply to the real world? ● How do circles and their parts relate to the physical world?
Enduring Understanding(s):	● Transformations are functions that can affect the measurements of a geometric figure. ● Congruent figures have equal corresponding angle measures and equal distances between corresponding pairs of points. ● Similar figures have equal corresponding angle measurements, and the distances between corresponding pairs of points are proportional. ● The geometry of a circle is completely determined by its radius.
Learning Goal(s): *Students will know and will be able to use their learning to:* (Content/ Skills)	**Content:** (Students will know/understand…) **<u>Transformations of Points in a Plane:</u>** ● Transformations describe motions in the plane. Analyzing these transformations indicates if and how these motions affect lengths and angle measures of figures. Congruence and Similarity are defined in terms of measurements that are preserved by transformations. ● A transformation is a function whose inputs and outputs are points in the plane. A set of all input points of a transformation is called a preimage; a set of all output points of the preimage is

	called an image. ● A rigid motion transformation preserves both the distance between pairs of points and the angle measures. A similarity transformation preserves angle measures but not necessarily between pairs of points. ● Given a transformation T and two points, A and B, the notation *T(A)=B* means that the image of point A under transformation T is point B. The transformation is said to map point A to point B. ● A rigid motion transformation is a transformation that preserves distances between pairs of points as well as angle measures. ● A translation is a transformation that maps each point in the plane to an image that is a specified distance in a specified direction from the preimage. ● A reflection is a transformation that maps each point in the plane to its mirror image across a line called the axis of symmetry. ● A rotation is a transformation that maps each point in the plane to an image that is turned by a specific angle about a fixed point called the center of rotation. ● Applying one or more translations, rotations, and reflections maps an object to a congruent object. ● Any transformation that preserves distance between points and angle measures can be written as a sequence of translations, reflections, and/or rotations. ● If two figures are congruent, there must exist a sequence of one or more rigid transformations that maps one figure to the other. ● A similarity transformation is a sequence of a dilation and/or one or more rigid motion transformations. ● A dilation from a fixed point, called the center, with a scale factor *k* is a transformation that maps each point in the plane to an image whose distance from the center is *k* times the distance between the center and the preimage, in the same direction as the preimage. ● Dilations of figures do not affect the angle measures of a figure. ● Dilating the plane by a scale factor *k* with center (0,0) will scale each coordinate by *k*. ● A dilation maps a line not passing through the center of the dilation to a parallel line and maps a line passing through the center of dilation to itself. ● The scale factor of a dilation can be determined by dividing the length from the image by its corresponding length in the preimage.

	● The perimeter of the image of a figure is the perimeter of the preimage scaled by the same scale factor as the dilation. **Congruence and Similar Polygons:** ● If three sides and three angles of a triangle are congruent to the three sides and three angles of another triangle, then the two triangles are congruent. ● If two triangles are congruent, then all six corresponding parts of the triangle are also congruent. ● If three sides of one triangle are congruent to three sides of another triangle, then the triangles are congruent (SSS). ● If two sides of a triangle and the interior angle they form are congruent to side sides of another triangle and the interior angle they form, then the triangles are congruent (SAS). ● If two angles and the side adjacent to both angles of a triangle are congruent to two angles and the side adjacent to both angles in another triangle, the triangles are congruent (ASA). ● If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and a leg of the other right triangle, then the two triangles are congruent (HL). ● Two triangles are similar if and only if they have two pairs of congruent angles. (AA ~). ● Two triangles are similar if and only if the lengths of their corresponding sides are in proportion (SSS~). ● Two triangles are similar if and only if one can be mapped to coincide with the other after applying a transformation(s). ● Proofs about parallelograms are based on relationships among their sides, angles, and diagonals. ● A line segment between opposite vertices in a parallelogram forms two congruent triangles that share a common side. ● For a parallelogram in the coordinate plane, the slopes of the sides and diagonals can be used to prove statements about the parallelogram. **Measurement of Lengths and Angles in Circles:** ● A point in the coordinate plane lies on a circle if its coordinates satisfy the equation of a circle. ● All points that lie on a circle are equidistant from the center of the circle. ● A circle is the set of all points equidistant from a given point. ● In the coordinate plane, the graph of the equation $(x - h)^2+(y - k)^2 = r^2$ is the set of all

points located r units from the points (h,k). This is a circle with radius r and center (h, k).

- A central angle is an angle whose vertex is the center of a circle and whose sides are, or contain, two radii of the circle.
- The measure of an arc is defined as the measure of the central angle that intercepts the arc.
- An inscribed angle is an angle whose vertex lies on a circle and whose sides contain chords of the circle.
- The measure of an inscribed angle is half the measure of the arc it intercepts. Equivalently, the measure of the intercepted arc is twice the measure of the inscribed angle.
- Inscribed angles that intercept the same ar have equal angle measures.
- The length of a circular arc depends on the measure of the central angle that intercepts the arc and the radius of the circle.
- The ratio of the length of a circular arc and the circumference of the circle is equal to the ratio of the measure of the central angle that intercepts the arc and the angle measure of a full circle.
- A line, ray, or line segment tangent to a circle intersects the circle at exactly one point.
- A line, ray, or line segment tangent to a circle is perpendicular to a radius of the circle at the point of intersection.
- In the coordinate plane, the slope of the line, ray, or line segment tangent to the circle and the slope of the radius that intersects this tangent line, ray, or line segment will be opposite reciprocals, or one will be vertical and the other will be horizontal.

Skills: (Students will be able to…)

- Perform Transformations on points in a plane.
- Express transformations using function notations.
- Prove that a rigid motion transformation maps an object to a congruent object.
- Solve problems involving rigid motion transformations.
- Prove that a similarity transformation maps an object to a similar object.
- Solve problems involving similarity transformations.
- Prove that two triangles are congruent by comparing their side lengths and angle measures.
- Prove that two triangles are congruent by comparing specific combinations of side lengths and angle measures.
- Prove that two triangles are similar.

	• Prove theorems about parallelograms. • Determine whether a particular point lies on a given circle. • Determine the intersection of a line and a circle. • Translate between the geometric and algebraic representations of a circle. • Prove that any two circles are similar. • Determine the measure of a central angle or the circular arc it intercepts. • Determine the measure of an inscribed angle or the circular arc it intercepts. • Determine the length of a circular arc. • Construct a line, ray, or line segment tangent to a circle.

Unit Number and Title:	Unit 3: Measurement in Two and Three Dimensions
Duration:	~ 4-6 weeks
Unit Overview:	This unit deepens students' understanding of measurement by expanding the concept of measurement to two dimensions through the areas of planar figures and to three dimensions through volumes of solid figures. One reason for studying area is that it often represents quantities that are otherwise difficult to compute. Therefore, techniques for calculating area can be adapted to find other quantities. Students use their prior experience with calculating the areas of conventional figures and composites of those figures, and their experience calculating the volumes of conventional solids. The unit introduces students to Cavalieri's principle, which relates the area of a figure to its cross-sectional lengths and the volume of a solid to its cross-sectional areas. The focus of the unit is on justifying area and volume formulas with which students are already familiar and using area and volume to model real-world physical scenarios. Throughout the course, specific learning objectives require students to prove geometric concepts. Students' proofs can be organized in a variety of formats, such as two-column tables or paragraphs. The format of a student's proof is not as important as their ability to justify a mathematical claim or provide a counterexample disproving one. They should develop an understanding that a mathematical proof establishes the truth of a statement by combining previously developed truths into a logically consistent argument.
Learning Goals	
Standard(s):	**GEOMETRY** **Geometric Measurement and Dimension (GMD)** **Explain volume formulas and use them to solve problems.** G.GMD.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments*. G.GMD.3

	Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. **Visualize relationships between two-dimensional and three dimensional objects.** G.GMD.4 Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects. **Expressing Geometric Properties with Equations (GPE)** **Use coordinates to prove simple geometric theorems algebraically.** G.GPE.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula. **Modeling with Geometry (MG)** **Apply geometric concepts in modeling situations.** G.MG.1 Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). G.MG.2 Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot). G.MG.3 Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). **NUMBER AND QUANTITY** **Quantities (Q)** **Reason quantitatively and use units to solve problems.** N.Q.1 Use units as a way to understand problems and to guide the solution of multi-step

	problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. N.Q.2 Define appropriate quantities for the purpose of descriptive modeling. N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. **Similarity, Right Triangles, and Trigonometry (SRT)** **Define trigonometric ratios and solve problems involving right triangles** G.SRT.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. **ALGEBRA** **Creating Equations (CED)** **Create equations that describe numbers or relationships** A.CED.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions*. A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales
Essential Question(s):	● How do geometric relationships and measurements help us to solve problems and make sense of our world? ● How does geometry model the physical world? ● How do mathematical ideas interconnect and build on one another to produce a coherent whole? ● How can a variety of appropriate strategies be applied in solving geometric problems?

	● How do the calculations and concepts of area and volume relate to two and three-dimensional objects?
Enduring Understanding(s):	● The area of a figure depends on its height and its cross-sectional widths. ● The volume of a solid depends on its height and its cross-sectional areas. ● The geometry of a sphere is completely determined by its radius.
Learning Goal(s): *Students will know and will be able to use their learning to:* (Content/ Skills)	**Content: (**Students will know/understand…) **<u>Area as a Two-Dimensional Measurement:</u>** ● If two figures have congruent bases and equal heights, and the line segments in the interiors of those figures that are parallel to, and equal distances from the base are congruent, then the figures will have equal area. ● The area of a sector depends on the measure of the central angle that forms the sector and the radius of the circle. ● The ratio of the area of a sector and the area of the circle is equal to the ratio of the measure of the central angle that forms the sector and the angle measure of a full circle. ● The area of the image of a figure is the area of the preimage scaled by the square of the scale factor of the dilation. (Linear/Area/Volume ratio relationship) **<u>Volume as a Three-Dimensional Measurement:</u>** ● The cross section of a right prism and pyramid is the polygon formed by the intersection of the solid with a plane parallel to its base. ● The volume of a right prism is equal to the product of the height of the solid and the area of its base. ● The volume of a pyramid is equal to one-third the product of the height of the solid and the area of its base. ● The cross section of a right cylinder and cone is the circle formed by the intersection of the solid with a plane parallel to its base. ● The volume of a right cylinder is equal to the product of the height of the solid and the area of its base.

	• The volume of a cone is equal to one-third the product of the height of the solid and the area of its base. • Physical objects in many real-world scenarios can be modeled by solid geometric figures such as prisms, pyramids, cylinders, and cones. **<u>Measurements of Spheres:</u>** • A sphere is an object in three-dimensional space that is the set of all points equidistant from a given point, called its center. Round physical objects in real-world scenarios can be modeled by spheres. • The surface area of a sphere is given by the formula $SA = 4\pi r^2$, where *r* represents the length of the radius of the sphere. • The volume of a solid sphere is given by the formula $V = \frac{4}{3}\pi r^3$. where *r* represents the length of the radius of the sphere. **Skills:** (Students will be able to…) • Use Cavalieri's principle to solve problems involving the areas and volume of figures. • Determine the area of a sector. • Determine the effect of a similarity transformation on the area of a figure. • Justify the volume formula for a right prism, pyramids, right cylinders, and cone. • Solve contextual problems involving volume of solid figures. • Define spheres in terms of distance. • Justify the surface area and volume formula for a sphere. • Solve contextual problems using spheres.

Unit Number and Title:	Unit 4: Measurement in Data
Duration:	~ 5-7 weeks
Unit Overview:	This unit offers a sustained and focused examination of statistics and probability to support the development of students' quantitative literacy. Statistics and probability help us perform essential real-world tasks such as making informed choices, deciding between different policies, and weighing competing knowledge claims. Students are expected to think about data sets as distributions which are functions that associate data values with their frequency or their probability. This encourages students to connect their knowledge of functions to concepts of statistics and probability, creating a more complete understanding of mathematics. Throughout the unit, students generate their own data through surveys, experiments, and simulations that investigate some aspect of the real world. They engage in statistical calculations and probabilistic reasoning as methods of analysis to make sense of data and draw inferences about populations. Incorporating statistics and probability in the same course as geometry allows students to experience two distinct forms of argumentation: geometrical reasoning as drawing conclusions with certainty about an ideal mathematical world, and probabilistic reasoning as drawing less-than-certain conclusions about the real world.
Learning Goals	
Standard(s):	**STATISTICS AND PROBABILITY** **Interpreting Categorical and Quantitative Data (ID)** **Summarize, represent, and interpret data on a single count or measurement variable.** S.ID.1 Represent data with plots on the real number line (dot plots, histograms, and box plots). S.ID.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. S.ID.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

S.ID.4
Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. ~~Use calculators, spreadsheets, and tables to estimate areas under the normal curve~~.

Making Inferences and Justifying Conclusions (IC)

Understand and evaluate random processes underlying statistical experiments.

S.IC.1
Understand statistics as a process for making inferences about population parameters based on a random sample from that population.

S.IC.2
Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?*

Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

S.IC.3
Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.

S.IC.4
Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.

S.IC.5
Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.

S.IC.6
Evaluate reports based on data.

Conditional Probability and the Rules of Probability (CP)

Understand independence and conditional probability and use them to interpret data.

S.CP.1

Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").

S.CP.2

Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.

S.CP.3

Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.

S.CP.4

Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.

S.CP.5

Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.

Use the rules of probability to compute probabilities of compound events in a uniform probability model

S.CP.6

	Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model. S.CP.7 Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.
Essential Question(s):	● How do we make predictions and informed decisions based on current numerical information? ● What are the advantages and disadvantages of analyzing data by hand versus by using technology? ● What is the potential impact of making a decision from data that contains one or more outliers?
Enduring Understanding(s):	● Statistics are numbers that summarize large data sets by reducing their complexity to a few key values that model their center and spread. ● Distributions are functions whose displays are used to analyze data sets. ● Probabilistic reasoning allows us to anticipate patterns in data. ● The method by which data are collected influences what can be said about the population from which the data were drawn, and how certain those statements are.
Learning Goal(s): *Students will know and will be able to use their learning to:* (Content/ Skills)	**Content: (**Students will know/understand…) **<u>The Shape of Data:</u>** ● Center of the data is a typical value in a data distribution. ● Measures of spread describe how similar or varied the set of observed values are for a particular variable (data item). ● Mean and median summarize a data distribution by identifying a typical value, or center, of the distribution. ● Standard deviation, interquartile range, and range summarize the distribution by quantifying the variability, or spread of the data set. The standard deviation, IQR and range have the same units as the values in the data distribution. ● Boxplots are used to depict the spread of the data. ● Histograms are used to depict the shape of a distribution and display frequency values.

	● Measures of center can be used to compare the typical values in a distribution; can provide information whether one distribution is typically larger, smaller or the same as another distribution. ● Mean is the center of mass of the data set; a weighted average. ● The mean is the only point in the domain of a distribution where the sum of the deviations, or differences, between the mean and each point in the distribution is zero. ● Symmetric distributions, such as the normal distribution, are the proportion of data to the left or right of the mean is equal. ● Skew describes the asymmetry of a distribution, the direction of the skew is indicated by the longer tail of values. ● In a skewed distribution the mean and median will be different, the farther apart the mean and median the more skewed the distribution will appear. ● Measures of variability quantify the typical spread of a data distribution; it is used to describe how similar the values of a data set are to each other. Low variability will have data values clustered at the center; high variability will have data values spread out from the center. ● The IQR is the length of the interval that contains 50% of the values in the distribution.. ● The normal distribution as a model of a data distribution defined by its mean and standard deviation. It is bell-shaped and symmetric about the mean. The frequency of data values tapers off at one standard deviation above or below the mean. ● For normally distributed data, the mean and median are the same number, and they correspond to the mode, which is the value in the distribution with the highest frequency. **<u>Chance Events:</u>** ● Venn diagrams and contingency tables (*two-way tables*) are common displays of categorical data and are useful for answering questions about probability. ● The intersection of the two categories is the set of elements common to both categories. ● The union of two categories is the set of elements found by combining all elements of both categories. ● For categorical data, variability is determined by comparing relative frequencies of categories. ● The sample space is the set of all outcomes of an experiment of random trial.

	● Probabilities are numbers between 0 and 1 where 0 means no possibility that an event can occur and 1 means the event is certain to occur. The probability of an event occurring can be described as the ratio of the number of favorable outcomes to the number of total outcomes in the sample space. ● The sum of the probabilities must be 1. ● Relative frequencies are the number of times an event occurs divided by the total number of observations. They can be used to estimate probabilities of future events occurring. ● Joint frequencies are events that co-occur for two or more variables; typically displayed in two-way contingency tables. ● Marginal frequencies are events that summarize the frequencies; they are the row totals and column totals in a two-way contingency table. ● The conditional probability of B, given A has already occurred, is the proportion of times B occurs when restricted to events only in A. ● Two events, A and B, are independent if the occurrence of A does not affect the probability of B. ● Two events, A and B, are independent if the probability of A and B occurring together is the product of their probabilities. **<u>Inferences from Data:</u>** ● Accuracy is how close the measurements are to the true value; determined by comparing the center of a sample of measurements to the true value of the measure. ● Precision is how close the measurements are to one another; determined by examining the variability of a sample measurements. ● Bias is the tendency to systematically overestimate or underestimate the true measure of a phenomenon. Bias is an indication of the inaccuracy of the measurement process. ● The law of large numbers states that the mean of the results obtained from a large number of trials will tend to become closer to the true value of the phenomenon being measured as more trials are performed; this means we trust larger samples more than smaller ones. ● The law of large numbers assumes there is no systematic error of measurement in the sample. ● An experiment is a method of gathering information about phenomena where the independent variable is manipulated by the researcher.

	- An observational study is a method of gathering information about phenomena where the independent variable is not under control of the researcher. - A survey is a method of gathering information from a sample of people using a questionnaire. - Experiments can be subject to systemic bias if the experiment does not sample from the population randomly and does not randomly assign sampling units to experimental and control conditions. - Observational studies can be subject to sampling bias if the sampling unit being observed is not randomly selected. - Surveys can be subject to bias from several factors, including sampling bias and response bias. **Skills:** (Students will be able to…) - Determine appropriate summary statistics for a data distribution. - Create a graphical representation of a data set. - Analyze data distribution with respect to their centers. - Analyze distribution with respect to their symmetry or direction of skew. - Analyze data distributions by their variability. - Model a data distribution with a normal distribution. - Create and analyze a data display for a categorical data set. - Determine the probability of an event. - Calculate relative frequencies, joint frequencies, marginal frequencies, or conditional probabilities for a categorical data set. - Determine if two events are independent. - For quantitative variables, calculate, compare, and interpret mean, median, and range. Interpret (but don't calculate) standard deviation. - Describe how the size of a sample impacts how well it represents the population from which it was drawn. - Design a method for gathering data that is appropriate for a given purpose. - Identify biases in sampling methods for experiments, observational studies, and surveys.