Grade(s):	**10-12**
Discipline/Course:	**Mathematics**
Course Title:	**Pre-Calculus**
Prerequisite(s):	**Algebra-2**
Course Description: *Program of Studies*	In this course, students will build upon their understanding of polynomials by exploring and understanding rational functions. Students will simplify rational expressions and use these understandings to solve rational equations. Additionally, students will explore the graphical representations of a rational function. Lastly, students will develop an understanding of a logarithm by building off their learnings about exponential functions and properties. Additionally, students will continue to develop an understanding of trigonometry that builds upon their right triangle trigonometry concepts from geometry. The course includes a strong emphasis on circular and triangular trigonometric functions, graphs of trigonometric functions and identities and trigonometric equations, and vectors. Furthermore, the conclusion of the course will end with an introduction to calculus with the investigation of limits.
Course Essential Questions:	***How can you use trigonometry to solve real world problems?*** ***How do function families behave and how can understanding the behaviors help us to make predictions and solve problems?***
Course Enduring Understandings:	● Real world situations can be represented symbolically and graphically. ● Algebraic expressions and equations generalize relationships from specific cases. ● Trigonometric functions are natural and fundamental examples of periodic functions.
Duration:	One Year
Course	Precalculus (Blizter Text)

Materials/Resources:	

***Note: Topics listed in the units may evolve over time based on adaptations to implementation. However, the overall content of the entire course will not change**

Algebra 2 Standards for Mathematical Practice The K-12 Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. This page gives examples of what the practice standards look like at the specified grade level. Students are expected to:	
Standards	Explanations and Examples
1. Make sense of problems and persevere in solving them.	Students solve problems involving equations and discuss how they solved them. Students solve real world problems through the application of algebraic and geometric concepts. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, “What is the most efficient way to solve the problem?”, “Does this make sense?”, and “Can I solve the problem in a different way?”
2. Reason abstractly and quantitatively.	This practice standard refers to one of the hallmarks of algebraic reasoning, the process of decontextualization and contextualization. Much of elementary algebra involves creating abstract algebraic models of problems and then transforming the models via algebraic calculations to reveal properties of the problems.
3. Construct viable arguments and critique the reasoning of others.	In Pre-Calculus, students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, graphs and tables. They further refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. They pose questions like “How did you get that?”, “Why is that true?” “Does that always work?” They explain their thinking to others and respond to others’ thinking.
4. Model with mathematics.	Indeed, other mathematical practices in Pre-Calculus might be seen as contributing specific elements of these two. The intent of the following set is not to decompose the above mathematical practices into component parts but to rather to show how the mathematical practices work together.
5. Use appropriate tools strategically.	Students consider available tools, graphing calculators and other technologies so they can strategically gain understanding of the ideas expressed by individual content standards and to model with mathematics.

6. Attend to precision.	In Pre-Calculus, the habit of using precise language is not only a mechanism for effective communication but also a tool for understanding and solving problems. Describing an idea precisely helps students understand the idea in new ways.
7. Look for and make use of structure.	In Pre-Calculus, converting rectangular equations describing the path of an object can be restructured into parametric form to allow the description of the objects position relative to horizontal position, vertical position, and time. A student can then tell not only the positions the object will occupy but also when it is in each position.
8. Look for and express regularity in repeated reasoning.	Creating equations or functions to model situations is harder for many students than working with the resulting expressions. An effective way to help students develop the skill of describing general relationships is to work through several specific examples and then express what they are doing with algebraic symbolism. For example, students should be able to see a pattern in the development of the points on the unit circle through their reasoning of 30-60-90 and 45-45-90 triangles.

Academic Expectations

The Fairfield Public Schools describe a variety of cross curricular expectations that all students should exemplify during their time within the schooling experience. This page gives examples of what the practice standards look like at the specified grade level. Students are expected to:

Standards	Explanations	Example
1. Exploring and Understanding	When students engage in problem solving situations, they should be able to understand the problem, determine relevant information, and ask relevant additional questions.	Students should be able to answer the following questions when approaching a problem: 1. Do you understand all the words used in stating the problem? 2. What are you asked to find or show? 3. Can you restate the problem in your own words? 4. Can you think of a picture or diagram that might help you understand the problem?
2. Synthesizing and Evaluating	Engaging in a problem solving situation, students should be able to analyze the most efficient approach, and reflect on the process used to solve the problem.	Students should be able to answer the following questions when analyzing how to approach a problem, and also reflect on the result: 1. Is there enough information to enable you to find a solution? If not, what additional information is needed? 2. Are there multiple ways to complete the task? Which approach do you think is most efficient, and why? 3. Do you know a related problem? Look at the unknown and try to think of a familiar problem having the same or similar unknown. Can you use it? 4. Was your strategy effective? What worked? What didn't? 5. Was there another approach that could have been more efficient? 6. Is your answer reasonable? How do you know? 7. Was your presentation approach effective? If not, what would you change? 8. How did the communication tools allow you to get the message across to the intended audience?

3. Creating and Constructing	Engaged in a problem solving situation, students should implement a plan.	Students should be able to answer the following question to implementing their plan to solve a problem: 1. What strategy will you use to complete the task?
4. Conveying Ideas	Students should be able to use correct mathematical language, logically display their work for the desired problem.	Students should be able to answer the following questions to convey their mathematical thinking to solve a problem: 1. How will you present your information to your intended audience? 2. Does your response illustrate the correct terms and work to the problem?
5. Using Communication Tools	Students should be able to choose the correct tools to illustrate their mathematical work to solve a specific problem.	Students should be able to answer the following question to use specific communication tools to solve a problem: 1. If applicable, what communication tools will you use to convey your ideas and solution?
6. Collaborating Strategically	Students should be able to work collaboratively to solve problems.	Students should be able to answer the following question to collaboratively solve problems: 1. In what ways did you work together to help solve the desired problem?

Unit Number and Title:	**Unit 1: Polynomial & Rational Functions and Equations**
Resource(s):	Blitzer Text: Chapter 2
Learning Goals	
Standard(s):	**NUMBER AND QUANTITY** **The Complex Number System** ***Use complex numbers in polynomial identities and equations.*** N-CN.8 (+) Extend polynomial identities to the complex numbers. For example, rewrite $x^2 + 4$ as $(x + 2i)(x–2i)$. N-CN.9 (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. **NUMBER AND QUANTITY** **Vector Quantities and Matrices** ***Perform arithmetic operations with complex numbers*.** N-CN.3 (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers. **ALGEBRA** **Arithmetic with Polynomials and Rational Expressions - A-APR** ***Rewrite rational expressions*** A-REI 11 Graph basic rational functions. A-REI 11

	Use graphs of rational functions and inequalities to solve problems. **FUNCTIONS** **Interpreting Functions - F-IF** ***Interpret functions that arise in applications in terms of the context.*** F-IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums ***Analyze functions using different representations*** F-IF.7 Graph functions expressed symbolically and show key features of the graph by hand in simple cases and using technology for more complicated cases. F-IF7d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. **ALGEBRA** **Arithmetic with Polynomials and Rational Expressions - A-APR** ***Rewrite rational expressions*** A-APR.7 (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions. **Reasoning with Equations and Inequalities - A-REI** ***Understand solving equations as a process of reasoning and explain the reasoning.*** A-REI 2

	Solve simple rational equations in one variable, and give examples showing how extraneous solutions may arise.
Essential Question(s):	● How can symmetry of a function be assessed? ● What are complex numbers, and how are these used to find non-real solutions? ● What are the graphical features of a polynomial and rational function that can be interpreted analytically and/or graphically? ● How can solution sets for polynomial and rational inequalities be determined from a graph? ● What are critical values and how are they used to determine solution sets to rational inequalities? ● What are the key characteristics of a rational function? ● How can a rational expression be re-expressed as a sum or difference of fractions? ● What strategies are used to solve rational equations? ● When do you know if a solution is extraneous?
Enduring Understanding(s):	● The fundamental theorem of algebra states that every non-constant single-variable polynomial with complex coefficients has at least one complex root. ● A rational function is a ratio of polynomial functions. If a rational function is in simplified form and the polynomial in the denominator is not a constant, the graph of the rational function features asymptotic behavior. ● To operate with rational expressions, you can use much of what you know about operating with fractions. 4. When solving an equation involving rational expressions multiplying by the common denominator can result in extraneous solutions. ● You can use much of what you know about multiplying and dividing fractions to multiply and divide rational expressions.
Learning Goal(s): *Students will be able to use their learning to:*	1. Perform operations with rational expressions including addition, subtraction, multiplication, division, and partial fraction decomposition to simplify rational expressions including complex rational expressions. 2. Solve rational equations using algebraic and graphical methods 3. Determine if a solution is extraneous by solving a rational equation

	4. Decompose a rational function into partial fractions including rational functions with irreducible quadratic roots in the denominator.
	5. Apply rational function models to solve problems (e.g., concentration or work problems).
	6. Identify the domain and range of a rational function
	7. Use key characteristics of the graph of a rational equation including intercept(s), asymptote(s), symmetry and end behavior to sketch the graph by hand in simple cases and using technology for more complicated cases.
	    a. Determine horizontal, vertical, and oblique asymptotes from a rational function.
	    b. Determine y- and x-intercepts
	    c. Determine holes within a rational function
	8. Use critical values and sign charts to solve and apply rational inequalities in real numbers.

Unit Number and Title:	**Unit 2: Inverses and Logarithms**
Resource(s):	Precalculus (Blitzer text): Chapter 3
Learning Goals	
Standard(s):	**FUNCTIONS** **Building Functions - BF** ***Build a function that models a relationship between two quantities*** F-BF.1c. (+) Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time. **Interpreting Functions F-IF** ***Analyze functions using different representations.*** F-IF 7e Graph logarithmic functions, showing intercepts and end behavior. **Building Functions F-BF** **Build new functions from existing functions** F-BF.4 Find inverse functions F-BF 4a Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. For example, $f(x) = 2x^3$ or $f(x) = \frac{x+1}{x-1}$ for $x \neq 1$. F-BF 4b (+) Verify by composition that one function is the inverse of another.

	F-BF 4c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse. F-BF 4d. (+) Produce an invertible function from a non-invertible function by restricting the domain. F-BF.5 (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents. F-BF.5a. Prove simple laws of logarithms. F-BF 5b. Use the definition of logarithms to translate between logarithms in any base. F-BF 5c. Understand and use the properties of logarithms to simplify logarithmic numeric expressions and to identify their approximate values.
Essential Question(s):	● What restricts a function from being invertible? ● Why does function composition determine if functions are inverses of each other? ● How are logarithmic and exponential functions related? ● How are the properties of logarithms connected to the properties of exponents? ● What are the properties of logarithms that allow us to re-write equivalent expressions or simplify? ● What is the shape of the graph of each function, how are they related and what does the end behavior describe? ● How can logarithms and exponential properties be utilized to solve "real world" problems?

Enduring Understanding(s):	● All exponential and logarithmic functions are transformations of a base function. ● Logarithms allow us to solve for exponential equations in which the variable is in the exponent. ● Logarithms have an inverse relationship with exponents. ● Functions with restricted domains can be combined to form a new function whose domain is the union of the functions to be combined as long as the function values agree for any input values at which the domains intersect. ● The inverse of a function is a function that reverses, or "undoes" the action of the original function. ● The graphs of a function and its inverse function are reflections across the line y = x.
Learning Goal(s): *Students will be able to use their learning to:*	1. Know the graph of the parent functions of both exponential and logarithmic functions with base *e* including shape and key characteristics including intercepts, symmetry and end behavior. 2. Know the properties of logarithms including condensing and expanding as simplifying strategies. 3. Apply the horizontal line test to determine whether a function is invertible. 4. Use composition to verify that functions are inverses of each other. 5. Use function composition to make new functions from other functions. 6. Determine if two functions are inverses of each other by composition. 7. Compare and contrast the characteristics of functions and their inverses, including one-to-oneness, domain, and range 8. Restrict a domain to make a function invertible and determine its inverse within the restricted domain. 9. Apply the inverse relationship of exponential and logarithmic equations to transform between the two forms. 10. Sketch the parent function graphs of the exponential and logarithmic functions with base e and determine intercepts, symmetry and end behavior. 11. Apply the condensing and expanding properties of logarithms and simplify expressions. 12. Solve a variety of logarithmic equations with and without the use of a calculator. 13. Apply logarithmic and exponential models to solve growth and decay situations such as banking, half-life and doubling time.

Unit Number and Title:	**Unit 3: Trigonometric Functions, Equations, Identities, and Proofs**
Resource(s):	Precalculus (Blitzer text): Chapters 4 & 5
Learning Goals	
Standard(s):	**FUNCTIONS** **Trigonometric Functions F-TF** ***Extend the domain of trigonometric functions using the unit circle.*** F-TF.7 (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context. *Prove and apply trigonometric identities* F-TF.9 (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.
Essential Question(s):	• How do you use the identities to simplify trigonometric expressions? • How do you find the exact value of an angle using trigonometric identities? • How do you use reasoning to prove (or disprove) trigonometric statements?
Enduring Understanding(s):	• An identity is a statement that is valid for all values of the variable for which the expressions in the equation are defined. • Trigonometric identities are valuable in a wide variety of contexts because they allow for expressions to be represented in more convenient forms.
Learning Goal(s): *Students will be able to use their learning*	1. Graph sine, cosine, tangent, secant, cosecant, and cotangent functions 2. Use the inverse trigonometric functions to determine an angle with a given trigonometric ratio.

to:	3. Use fundamental trigonometric identities to evaluate trigonometric functions and simplify trigonometric expressions 4. Verify trigonometric identities 5. Use standard algebraic techniques and inverse trigonometric functions to solve trigonometric equations 6. Use sum and difference formulas, and multiple angle formulas, power-reducing formulas, half-angle formulas, and product-to-sum and sum-to-product 7. Rewrite and evaluate trigonometric functions

Unit Number and Title:	**Unit 4: Trigonometric Applications, Vectors, and Polar Functions**
Resource(s):	Precalculus (Blitzer text): Chapter 6
Learning Goals	
Standard(s):	**GEOMETRY** **Similarity, Right Triangles, and Trigonometry** ***Apply trigonometry to general triangles*** G-SRT.9 (+) Derive the formula $A = \frac{1}{2}absinC$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. G-SRT.10 (+) Prove the Laws of Sines and Cosines and use them to solve problems. G-SRT.11 (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces). **NUMBER AND QUANTITY** **Vector Quantities and Matrices** ***Represent and model with vector quantities.*** N-VM.1 (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., v, |v|, ||v||, v). N-VM.2 (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.

	N-VM.3 (+) Solve problems involving velocity and other quantities that can be represented by vectors. ***Perform operations on vectors.*** N-VM.4 (+) Add and subtract vectors. a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. c. Understand vector subtraction v – w as v + (–w), where –w is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise. N-VM.5 (+) Multiply a vector by a scalar. a) Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as c(vx, vy) = (cvx, cvy). b) Compute the magnitude of a scalar multiple cv using ||cv|| = |c|v. Compute the direction of cv knowing that when |c|v ≠ 0, the direction of cv is either along v for (c > 0) or against v (for c < 0). N-CN.4 (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.
Essential Question(s):	• How can we use trigonometric laws and formulas to solve problems? • How do you know when to use the Law of Sines and Cosines? • When will I use vectors to model physical quantities and solve problems?

Enduring Understanding(s):	● Trigonometric laws can be used to solve problems with non-right triangles ● Vectors can be utilized to display the magnitude and direction of an object in a physical application.
Learning Goal(s): *Students will be able to use their learning to:*	1) Use Law of Sines and Cosines to solve oblique triangles and determine area 2) Establish and use trigonometric laws in real world situations to solve problems (e.g., angles of elevation/depression, distance between two locations, etc.) 3) Differentiate from given measurements in triangles whether it is appropriate to use the Law of Sines or Cosines 4) Find missing measures of triangles (acute and oblique triangles) from given measurements. 5) Explain why the Law of Sine produces two possible cases from SSA given measurements. 6) Find the area of figures and applied problems using $A = \frac{1}{2}ab \cdot \sin C$ or Heron's formula. 7) Use appropriate symbols for vectors and their magnitudes 8) Find the components of a vector 9) Add vectors using a variety of techniques such as graphing them end-to-end, using their components, and/or using the parallelogram rule 10) Represent vector subtraction graphically by connecting the tips in the appropriate order and using their components 11) Determine the magnitude and direction of the sum of two vectors given the magnitude and direction of each 12) Compute the magnitude of a scalar multiple cv using $\|cv\| = \|c\|v$ 13) Describe the direction of cv knowing that when $\|c\|v \neq 0$, the direction of cv is either along v (for $c > 0$) or against v (for $c < 0$) 14) Use vectors to model velocities and forces and use them to solve problems 15) Plot rectangular points in the polar coordinate system, and vice versa. 16) Convert an equation from rectangular to polar form, and vice versa 17) Use point plotting to graph polar equations

Unit Number and Title:	**Unit 5: Conic Sections & Parametric Functions and Equations (honors only)**
Resource(s):	Precalculus (Blitzer): Chapter 9
Learning Goals	
Standard(s):	**GEOMETRY** **Geometric Properties and Equations - GPE** ***Translate between the geometric description and the equation for a conic section*** G.GPE.2 Derive the equation of a parabola given a focus and directrix G.GPE.3 (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.
Essential Question(s):	● How can the understanding of conic sections make more sense of the constructions and designs in our world? ● What determines the type of conic section you will be using? ● Why are there key vital coordinates, points and axes and how do they help me use and apply the conic section to solve problems? ● What does a complex number represent in a Cartesian plane? ● How can we find a parametric equation for a line segment, given endpoints? ● How do parametric equations and vectors solve real-world problems? ● How can different forms of an equation help one understand different properties of the function (rectangular vs. polar functions)?

Enduring Understanding(s):	● Conic sections are used in a variety of construction and scientific applications such as bridge design, planetary and satellite orbits, mirrors, navigation and arches. ● The angle or direction you cut through a cone determines the type of conic you will get. ● Certain key points and axes are vital to graphing and using the conic sections to real world problems. ● A conic section can be classified based on its general equation. ● Parametric equations provide more information about mathematical relations over time. ● Parametric equations allow analysis of 2-dimensional position and time.
Learning Goal(s): *Students will be able to use their learning to:*	1. Write the equation and describe the characteristics of a conic section given its graph. 2. Represent conic sections algebraically and graphically. 3. Describe the properties of circles, ellipses, hyperbolas, and parabolas. 4. Know that parametric equations can be used to model the path of an object 5. Evaluate sets of parametric equations for given values of the parameter 6. Convert parametric equations to rectangular form eliminating the parameter 7. Create a parametric equation set from a rectangular equations 8. Sketch/graph parametric equations 9. Solve problems using parametric equations 10. Use vectors and parametric equations to represent the movement of an object with constant velocity 11. Describe a path and key features of the path by giving the x and y coordinates of the path as functions of a third parameter such as time.

Unit Number and Title:	**Unit 6: Introduction to Limits**
Resource(s):	Precalculus (Blitzer): Chapters 11
Learning Goals	
Standard(s):	N/A
Essential Question(s):	● How do limits describe the behavior of a function? ● What are the strategies used to determine the limit of a function? ● What determines continuity and how can you find and describe discontinuities?
Enduring Understanding(s):	● A concept of a limit allows you to determine the value of a function by getting really close to a specified value ● A function's continuity at a given point affects its limit
Learning Goal(s): *Students will be able to use their learning to:*	1. Understand limit notation 2. Find limits using tables 3. Find limits using graphs 4. Find one-sided limits and use them to determine if a limit exists 5. Find limits of constant functions and identify the function 6. Find limits using properties of limits 7. Find one-sided limits using properties of limits 8. Find limits of fractional expressions in which the limit of the denominator is zero 9. Determine whether a function is continuous at a number 10. Determine for what numbers a function is discontinuous