

Foundations of Algebra

Course Information

Grade(s):	9
Discipline/Course:	Mathematics / Foundations of Algebra
Course Title:	Foundations of Algebra
Prerequisite(s):	N/A
Course Description: <i>Program of Studies</i>	This course is designed for students who did not successfully complete a pre-algebra course and would benefit from building their algebra foundational skills. . Building on their work with expressions and equations from Pre-Algebra within middle school, students in Foundations of Algebra will extend their skills to inequalities, linear equations, functions, exponent properties, systems of linear equations, and variable expressions. In the end, students will apply their mathematical learning to real-world problems and situations.
Course Essential Questions:	<ul style="list-style-type: none"> • How are quantitative relationships represented by numbers? • What is a proportional relationship and how can proportional relationships help us to solve problems? • How do I use algebraic expressions to analyze or solve problems? • What is the relationship between solving problems and computation? • Why is the ability to solve problems the heart of mathematics?
Course Enduring Understandings:	<ul style="list-style-type: none"> • Algebraic expressions and equations generalize relationships from specific cases. • Quantitative relationships can be expressed numerically in multiple ways in order to make connections and simplify calculations using a variety of strategies, tools and technologies. • Real world situations can be represented symbolically and graphically.
Duration: Credit:	One Year 1.0 Credit(s)
Course Materials/Resources:	EdGems
FPS Course Academic Expectation(s):	Exploring and Understanding Synthesizing and Evaluating

Year at a Glance (Units):	Unit 1: Expressions (~ 3-4 weeks) Unit 2: Equations and Inequalities (~ 6-8 weeks) Unit 3: Functions (~ 6-8 weeks) Unit 4: Systems of Equations (~ 4-6 weeks) Unit 5: Exponent Properties (~ 3-4 weeks) Unit 6: Application of Algebraic Concepts (~ 3-4 weeks)
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Unit Number and Title:	Unit 1: Expressions
Duration:	~ 3-4 weeks
Unit Overview:	In the first unit of the course, students build upon their previous experience with number systems (focused on rational numbers) and expressions by performing operations on expressions, recognizing equivalent expressions and simplifying. The focus of this unit is to strengthen students' numeracy skills and apply these skills to expression manipulation. Additionally, this unit will have a focus on the mathematical language that will be used throughout Algebra, recognizing equivalent forms of numbers and expressions, and categorizing values and expressions. Throughout this course and subsequent courses, fluency with numbers and expressions is a foundation for all other algebra skills.
Learning Goals	
Standard(s):	<p>Number System (NS)</p> <p>Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.</p> <p>7.NS.1</p> <p>Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.</p> <ol style="list-style-type: none"> Describe situations in which opposite quantities combine to make 0. For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged. Understand $p + q$ as the number located a distance q from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. Apply properties of operations as strategies to add and subtract rational numbers. <p>Know that there are numbers that are not rational, and approximate them by rational numbers.</p>

8.NS.1

Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.

ALGEBRA

Seeing Structure in Expressions (SSE)

Write expressions in equivalent forms to solve problems.

A.SSE.1

Interpret expressions that represent a quantity in terms of its context.

a. Interpret parts of an expression, such as terms, factors, and coefficients.

A.SSE.2

Use the structure of an expression to identify ways to rewrite it.

A.SSE.3

Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

NUMBER AND QUANTITY

Quantities (Q)

Reason quantitatively and use units to solve problems.

N.Q.1

Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

Essential Question(s):

- How can we classify numbers?
- How can any sum, difference, product or quotient of two rational numbers be restated as an equivalent statement?
- How can we use an expression to represent a quantity?
- What is the correct order for performing mathematical operations?

	<ul style="list-style-type: none"> • How can expressions be written to indicate an order for operations? • How does changing the order of operations affect the outcome when simplifying an expression?
Enduring Understanding(s):	<p>Students will understand that</p> <ul style="list-style-type: none"> • The Real Number System is organized into subcategories of numbers with similar traits. • There is a specific order of operations in the real number system that must be followed for all computations. • An expression is made up of variables and constants, along with algebraic operations. • Algebra uses symbols to represent quantities that are unknown or that vary. Mathematical phrases and real-world relationships can be represented using symbols and operations. • An algebraic expression can be simplified by combining the parts of the expression that are alike.
Learning Goal(s): <i>Students will be able to use their learning to:</i>	<p>Content: (Students will know/understand...)</p> <ul style="list-style-type: none"> • Numbers can be classified as rational or irrational. • Rational numbers also include natural numbers, whole numbers, and integers. • Expressions are used to represent a quantity. • Expressions contain numbers, variables, and at least one operation. • The expressions on either side of the equal sign are equivalent. • Order of operations is a rule that tells the correct sequence of steps for evaluating an expression. • The distributive property results in equivalent expressions. • Combining like terms is a technique for simplifying algebraic expressions. • Parts of an expression include terms, factors, and coefficients. <p>Skills: (Students will be able to...)</p> <ul style="list-style-type: none"> • Add, subtract, multiply, and divide rational numbers. • Identify whether a number is rational or irrational. • Simplify variable expressions. • Identify the parts of an expression including variable, coefficient, and constant. • Apply properties to generate equivalent expressions. • Apply the distributive property to an expression. • Use order of operations to simplify expressions.

Unit Number and Title:	Unit 2: Equations and Inequalities
Duration:	~ 6-8 weeks
Unit Overview:	In this unit, students will continue to develop the language of mathematics by distinguishing between an expression and an equation and interpret an equation as an equality of two expressions. They will apply their understanding of balancing an equation to solve multi-step linear equations with variables on both sides of the equals sign. When solving linear equations, students will work with equations that have one solution, no solution or infinitely many solutions. Students will extend the skill of solving a multi-step equation to solving multi-step inequalities with variables on both sides of the inequality sign. Solutions to inequalities will be represented on a number line.
Learning Goals	
Standard(s):	<p>Expressions and Equations (EE) Analyze and solve linear equations and pairs of simultaneous linear equations. 8.EE.7 Solve linear equations in one variable</p> <ol style="list-style-type: none"> Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers). Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms. <p>ALGEBRA Reasoning with Equations and Inequalities (REI) Understand solving equations as a process of reasoning and explain the reasoning. A.REI.1</p>

	<p>Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.</p> <p>Solve equations and inequalities in one variable. A.REI.3 Solve linear equations and inequalities and inequalities in one variable, including equations with coefficients represented by letters.</p> <p>NUMBER AND QUANTITY Quantities (Q) Reason quantitatively and use units to solve problems. N.Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.</p>
Essential Question(s):	<ul style="list-style-type: none"> • How is an expression different from an equation? • How do you balance an equation/inequality to solve for a variable? • What are inverse operations? • How can we solve multi-step equations and inequalities? • What does it mean when linear equations have one solution? No solutions? Infinitely many solutions? • How can you determine if a value is a solution to an equation/inequality?
Enduring Understanding(s):	<p>Students will understand that</p> <ul style="list-style-type: none"> • An equation is a mathematical statement that shows that two mathematical expressions are equal. The equal sign indicates that two expressions are equivalent. • One variable equations are the foundation for Algebra and solving them is rooted in mathematical properties (distributive property, addition/subtraction property of equality, multiplication/division property of equality). • Variables represent an unknown numerical value.

	<ul style="list-style-type: none"> • An inequality is another way to describe a relationship between expressions; instead of showing that the values of two expressions are equal, inequalities indicate that the value of one expression is greater than (or greater than or equal to) the value of the other expression. • In solving an inequality, multiplying or dividing both expressions by a negative number reverses the sign that indicates the relationships between the two expressions.
Learning Goal(s): <i>Students will be able to use their learning to:</i>	<p>Content: (Students will know/understand...)</p> <ul style="list-style-type: none"> • The difference between an equation and an expression is that an expression does not include an equal sign, while an equation does. • The difference between simplifying and solving is that simplifying involves making an expression shorter while solving involves finding the value(s) that make an equation true. Simplifying can be done with either expressions or within equations. • Inverse operations are used to solve equations. • The properties of equality are used for balancing an equation and an inequality. • The solution set of an inequality is the numbers that make an inequality statement true. • Linear equations can have one solution, no solutions, or infinitely many solutions. <p>Skills: (Students will be able to...)</p> <ul style="list-style-type: none"> • Solve one-step, two-step, and multi-step equations. • Solve with equations that have one solution, no solution or infinitely many solutions. • Apply their understanding of balancing an equation to solve multi-step linear equations with variables on both sides of the equal sign. • Solve one-, two-, and multi-step inequalities. • Solve inequalities and graph their solutions on a number line. • Apply their understanding of balancing an inequality to solve multi-step linear inequalities with variables on both sides of the inequality sign. • Verify a solution to an equation/inequality.

Unit Number and Title:	Unit 3: Functions
Duration	~ 6-8 weeks
Unit Overview:	<p>In this unit, students will build upon the concept of a function. Students will determine if a relationship is a function by examining a graph or table and examine a specific type of function formed by a proportional relationship. Students will then complete an in-depth study of linear functions as well as look at a variety of non-linear functions. Students will also describe qualitative features of a graph such as increasing, decreasing, linear and nonlinear.</p> <p>When working with linear functions, students will graph linear functions from an equation as well as write equations for linear functions based on graphs or key information. Students will understand that linear functions can be written in different forms (slope-intercept, point slope and standard form). The main goal of this course is building proficiency with slope-intercept form. Students interpret the constant of proportionality (unit rate) as the slope of the graph and progress from understanding the slope as the unit rate to calculating slope from a graph, a table or two ordered pairs.</p>
Learning Goals	
Standard(s):	<p>Number System (NS) Apply and extend previous understandings of numbers to the system of rational numbers. 6.NS.8 Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.</p> <p>Ratios and Proportional Relationships (RP) Analyze proportional relationships and use them to solve real-world and mathematical problems. 7.RP.2 Recognize and represent proportional relationships between quantities. a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.</p>

- b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
- c. Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p , the relationship between the total cost and the number of items can be expressed as $t = pn$.

Functions (F)

Define, evaluate, and compare functions.

8.F.1

Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.

8.F.2

Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.

8.F.3

Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.

Use functions to model relationships between quantities.

8.F.4

Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

8.F.5

Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

FUNCTIONS

Interpreting Functions (IF)

Understand the concept of a function and use function notation.

F.IF.1

Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If it is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x . The graph of f is the graph of the equation $y = f(x)$.

F.IF.2

Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Interpret functions that arise in applications in terms of the context.

F.IF.4

For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

F.IF.5

Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.

F.IF.7

	<p>Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.</p> <p>a. Graph linear and quadratic functions and show intercepts, maxima, and minima.</p> <p>Interpret functions that arise in applications in terms of the context.</p> <p>F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.</p> <p>Linear, Quadratic, and Exponential Models (LE)</p> <p>Interpret expressions for functions in terms of the situation they model.</p> <p>F.LE.5. Interpret the parameters in a linear or exponential function in terms of a context.</p> <p>ALGEBRA</p> <p>Reasoning with Equations and Inequalities (REI)</p> <p>Represent and solve equations and inequalities graphically.</p> <p>A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).</p> <p>Creating Equations (CED)</p> <p>Create equations that describe numbers or relationships.</p> <p>A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear functions.</p> <p>A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.</p>
Essential Question(s):	<ul style="list-style-type: none"> What is the relationship that describes a function?

	<ul style="list-style-type: none"> • How can functions be used to find the output from a given input? • How can equations, graphs, word descriptions, and tables describe a function? • What is a rate and how is it related to proportional reasoning?
Enduring Understanding(s):	<p>Students will understand that...</p> <ul style="list-style-type: none"> • Functions are single-valued mappings from one set - the domain of the function - to another - its range. • A function's rate of change is one of the main characteristics that determine what kinds of real-world phenomena the function can model. • Linear functions are characterized by a constant rate of change. Reasoning about the similarity of "slope" triangles allows deducing that linear functions have a constant rate of change and a formula of the type $y = mx + b$. • Functions can be represented in various ways, including through algebraic means (e.g., equations), graphs, word descriptions, and tables.
Learning Goal(s): <i>Students will be able to use their learning to:</i>	<p>Content: (Students will know/understand...)</p> <ul style="list-style-type: none"> • Ratios that are equal to each other form a proportional relationship. The equation stating that two ratios are equal is a proportion. • The unit rate for each ratio in a proportional relationship is constant. This rate is called the constant of proportionality. For proportional relationships, this is represented by m in the equation $y = mx$. This rate of change is also known as slope. • The slope is a unit rate of change in a proportional relationship. • A function is a rule that assigns each input to exactly one output. • The vertical line test can be used to see if a graph represents a function. (i.e., if the vertical line intersects a graph in more than one point, that means there is at least one x-value that has more than one output value, then the graph is not a function.) • The difference between a linear and a nonlinear function by examining their graphs. Linear functions are functions whose graphs form a straight line. • All linear functions increase or decrease at a constant value which is referred to as the unit rate or rate of change. • The domain of a function is the set of all input values. The range of a function is the set of output values. • Slope is the ratio of the vertical change in y-values (the rise) to the horizontal change x-values (the run). rise/run

- The y-intercept is the value of y where the graph crosses the y-axis and has an x-value of 0, and is also referred to as the initial value of a function.
- Proportional relationships are linear functions that go through the origin and have a constant slope (or rate of change), and have a y-intercept of 0.
- Linear equations can be written in slope-intercept form, standard form and point-slope form.
- Slope-Intercept Form gives both the slope and the y-intercept of the graph.

Skills: (Students will be able to...)

- Determine if a relationship is a function by examining a graph or table.
- Examine a specific type of function formed by a proportional relationship.
- Interpret the constant of proportionality (unit rate) as the slope of the graph.
- Interpret the slope as the unit rate in contextual problems.
- Calculate slope from a graph, a table or two ordered pairs.
- Describe qualitative features of a graph such as increasing, decreasing, linear and nonlinear.
- Plot ordered pairs on a coordinate plane.
- Identify and label quadrants on a coordinate plane.
- Write a linear equation in slope-intercept form from a graph, a table, or key information.
- Graph linear functions from an equation (in slope-intercept form), table, or key information.
- Determine the domain and range of a function from an equation, a table, or a graph.

Unit Number and Title:	Unit 4: Systems of Equations
Duration:	~ 4-6 weeks
Unit Overview:	Across this unit, students will solve systems of equations in support of two goals: to determine the solution to the system of equations and to become strategic and efficient in choosing the method to solve the system. Students will be able to differentiate between one solution set, no solutions or infinitely many solutions both from a graphical representation and from equations. Through these contexts students build upon their knowledge of solving equations and develop more sophisticated understandings about what the solution(s) to a system means in the context of the problem.
Learning Goals	
Standard(s):	<p>Expressions and Equations (EE) Analyze and solve linear equations and pairs of simultaneous linear equations. 8.EE.8 Analyze and solve pairs of simultaneous linear equations.</p> <ol style="list-style-type: none"> Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. <p>ALGEBRA Creating Equations (CED) Create equations that describe numbers or relationships. A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.</p> <p>Reasoning with Equations and Inequalities (REI) Understand solving equations as a process of reasoning and explain the reasoning.</p>

	<p>A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.</p> <p>Solve systems of equations.</p> <p>A.REI.5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.</p> <p>A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.</p>
Essential Question(s):	<ul style="list-style-type: none"> • How can systems of linear equations be solved by graphing? • How can systems of linear equations be solved algebraically? • What does it mean when the graphs of two linear functions intersect, are parallel, or overlap? • What method is most appropriate to solve a system of equations (graph, substitution, or elimination)?
Enduring Understanding(s):	<p>Students will understand that...</p> <ul style="list-style-type: none"> • A solution to a system of linear equations is an ordered pair of numbers that satisfies all the equations simultaneously. • Solving a system of linear equations is a process of determining the value or values that make the equation true. • Solving a system of equations can be computed graphically or algebraically. • The different methods to solve a system of equations can be more efficient than others, based on the situation and context.
Learning Goal(s):	<p>Content: (Students will know/understand...)</p> <ul style="list-style-type: none"> • A solution to a system of linear equations, if one exists, is an intersection point of the lines corresponding to the equations. • A solution to a system of linear equations, if one exists, corresponds to the solutions that the equations have in common.

Students will be able to use their learning to:

- A system of two linear equations can have no solution, one solution, or infinitely many solutions.
- If the graphs of two linear equations in a system are parallel, then the system has no solutions. If the graphs of two linear equations are not parallel and do not coincide, then the system has one solution. If the graphs of two linear equations coincide, then the system has infinitely many solutions.
- Algebraic methods of solving a system of equations include the substitution method and the elimination method.
- An efficient method of solving a system of equations should be based on the forms of the equations in the system.

Skills: (Students will be able to...)

- Use a graph or tables of values to determine the solution to a system of equations.
- Determine the number of real solutions to a system of two linear equations.
- Apply their knowledge of solving equations and graphing lines to solve systems of equations.
- Solve a system of linear equations using both graphing and algebraic methods.
- Look at systems of two linear equations and determine whether the lines are parallel, intersecting, or the same line.
- Determine the best method for solving a system of equations.

Unit Number and Title:	Unit 5: Exponent Properties
Duration:	~ 3-4 weeks
Unit Overview:	In this unit, students will apply their understanding of exponents to learn properties of exponents and create equivalent expressions. Students will learn multiplication and division properties of exponents. They will extend these properties to exponents of 0 and negative exponents. Students will work with expressions and equations with exponents in the variable (squares and cubes only). Prior to solving equations with degree greater than one, students revisit perfect squares and perfect cubes as well as square roots and cube roots. Finally, students will simplify square roots to represent irrational numbers in simplest form.
Learning Goals	
Standard(s):	<p>Expression and Equations (EE) Work with radicals and integer exponents. 8.EE.1 Know and apply the properties of integer exponents to generate equivalent numerical expressions. 8.EE.2 Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes.</p> <p>ALGEBRA Seeing Structure in Equations (SEE) Write expressions in equivalent forms to solve problems. A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.</p>

	<p>a. Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%</p>
Essential Question(s):	<ul style="list-style-type: none"> What are the components of an expression with an exponent greater than one (coefficient, variable, base, exponent)? How can properties of exponents be utilized to simplify expressions? How can you use properties of exponents to create equivalent expressions? How can you solve an equation with a square or cube on a variable?
Enduring Understanding(s):	<p>Students will understand that...</p> <ul style="list-style-type: none"> There are specific rules for how to simplify expressions involving exponents. Properties of exponents are derived from the properties of multiplication and division. Finding a root of a number is the inverse operation to the corresponding exponent. Powers can be used to shorten the representation of repeated multiplication.
Learning Goal(s): <i>Students will be able to use their learning to:</i>	<p>Content: (Students will know/understand...)</p> <ul style="list-style-type: none"> Exponential expressions involving multiplication can be rewritten by invoking the product rule; $n^a \cdot n^b = n^{a+b}$, where $n > 0$. Exponential expressions involving division can be rewritten by invoking the quotient rule; $\frac{n^a}{n^b} = n^{a-b}$, where $n > 0$. Exponential expressions involving powers of powers can be rewritten by invoking the power of a power rule; $(n^a)^b = n^{a \cdot b}$, where $n > 0$. Exponential expressions involving powers of powers can be rewritten by invoking the power of a product rule; $(mn)^a = m^a \cdot n^a$, where $m > 0$ and $n > 0$. Exponential expressions involving powers of powers can be rewritten by invoking the power of a quotient rule; $(\frac{m}{n})^a = \frac{m^a}{n^a}$, where $m > 0$ and $n > 0$. Any non-zero real number raised to the zero power is equal to 1. The zero exponent rule: $n^0 = 1$, where

$n \neq 0$.

- Zero raised to the zero power is not defined in the real number system.
- A negative exponent of -1 can be used to represent a reciprocal. The negative exponent rule: $n^{-k} = \frac{1}{n^k}$, where $n \neq 0$.
- The properties of negative integer exponents, and those of an exponent of zero, are extensions of the properties of positive integer exponents.
- The value of an irrational number cannot be expressed exactly as a ratio of integers or as non-repeating, non-terminating decimal, and is often represented exactly by a symbol such as π or $\sqrt{2}$.
- The square root of a squared real number is equivalent to the absolute value of the number; $\sqrt{a^2} = |a|$
- The cube root of a cubed real number is equivalent to the value of the number; $\sqrt[3]{a^3} = a$.

Skills: (Students will be able to...)

- Apply properties of exponents and create equivalent expressions.
- Apply the product rule, quotient rule, power of a power rule, power of a product rule, and power of a quotient rule for exponents.
- Apply their understanding of 0 and negative exponent values to create equivalent numerical expressions
- Decide which rule(s) to use to simplify an expression with exponents.
- Work with equations with exponents on the variable (squares and cubes).
- Find perfect squares and perfect cubes as well as square roots and cube roots.
- Simplify square roots to represent irrational numbers in simplest form.

Unit Number and Title:	Unit 6: Application of Algebraic Concepts
Duration:	~ 3-4 weeks
Unit Overview:	In this culminating unit, students apply the skills and processes from all of the previous units to solve problems in the real world. Students will learn how technology can be used within the mathematical process to solve these applications. Applications will include work with percentages, rates, unit rates, proportions, linear growth and decay and right triangle distances. Students will use systems of linear equations to model problems and develop more sophisticated understandings about what the solution(s) to a system means in the context of the problem and real-world situations.
Learning Goals	
Standard(s):	<p>Number System (NS) Apply and extend previous understandings of numbers to the system of rational numbers. 6.NS.8 Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.</p> <p>Ratios and Proportional Relationships (RP) Analyze proportional relationships and use them to solve real-world and mathematical problems. 7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks $\frac{1}{2}$ mile in each $\frac{1}{4}$ hour, compute the unit rate as the complex fraction $\frac{\frac{1}{2}}{\frac{1}{4}}$ miles per hour, equivalently 2 miles per hour.</p> <p>Expression and Equations (EE) Solve real-life and mathematical problems using numerical and algebraic expressions and equations.</p>

7.EE.3

Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

7.EE.4

Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

- a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p , q , and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.
- b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p , q , and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.

Work with radicals and integer exponents.

8.EE.2

Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes.

Analyze and solve linear equations and pairs of simultaneous linear equations.

8.EE.8

Solve linear equations in one variable.

- e. Solve real-world and mathematical problems leading to two linear equations in two variables.

Geometry (G)

Understand and Apply Pythagorean Theorem.

8.G.7

Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

NUMBER AND QUANTITY

Quantities (Q)

Reason quantitatively and use units to solve problems.

N.Q.1

Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

N.Q.2

Define appropriate quantities for the purpose of descriptive modeling.

N.Q.3

Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

ALGEBRA

Creating Equations (CED)

Create equations that describe numbers or relationships.

A.CED.1

Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear functions.

A.CED.2

Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

A.CED.3

Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.

Linear, Quadratic, and Exponential Models (LE)

Interpret expressions for functions in terms of the situation they model.

F.LE.5.

Interpret the parameters in a linear or exponential function in terms of a context.

Essential Question(s):	<ul style="list-style-type: none"> • How does the order of operations affect an outcome? • How does the Pythagorean Theorem help solve real world problems? • How can an algebraic equation be used to solve percent problems? • What are relationships that can be modeled as linear functions? • How can systems of equations be used to represent and solve real-world situations?
Enduring Understanding(s):	<p>Students will understand that...</p> <ul style="list-style-type: none"> • Real world situations can be represented symbolically and graphically. • Algebraic expressions and equations can be used to help solve real-world problems. • Right triangle distances can be determined by applying Pythagorean Theorem. • Systems of linear equations can be used to model scenarios that include multiple constraints.
Learning Goal(s): <i>Students will be able to use their learning to:</i>	<p>Content: (Students will know/understand...)</p> <ul style="list-style-type: none"> • Real-world scenarios can be modeled by linear functions. • A system of linear equations can be used to determine when two linear functions that model a contextual scenario have the same input–output pair. • A system of linear equations can be used to model a contextual scenario in which two quantities are subject to multiple constraints. • A system of linear equations derived from a contextual scenario can be solved free of context, but the solution must be interpreted in context to be correctly understood. • Model real world situations by using the Pythagorean Theorem. <p>Skills: (Students will be able to...)</p> <ul style="list-style-type: none"> • Use mathematical problem solving skills effectively. • Make decisions and solve problems in independent and collaborative settings. • Apply knowledge of decimals, fractions, and percents to solve real world problems. • Translate between words and mathematical expressions or equations. • Analyze a real world application of a linear function. • Model a contextual scenario with a system of linear equations. • Use the Pythagorean Theorem to solve real life problems.