Algebra II Honors

Course Information

Grade(s):	9-12
Discipline/Course:	Mathematics / Algebra II
Course Title:	Algebra II H
Prerequisite(s):	Successful Completion of one of the following: Algebra I Algebra I H May take concurrently with Geometry, with permission.
Course Description: *Program of Studies*	Building on their work with linear and quadratic functions from Algebra I, students in Algebra II will extend their repertoire of functions to include other parent functions with a focus on polynomial, exponential, logarithmic, and trigonometric functions. Students work closely with the expressions that define the functions and continue to expand and hone their abilities to model situations. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations. The course has additional content standards beyond the Algebra 2 course as well as an increased focus on rigor and depth of study. Strong algebra skills are required.
Course Essential Questions:	How do function families behave and how can understanding the behaviors help us to make predictions and solve problems?
Course Enduring Understandings:	Real world situations can be represented symbolically and graphically. Algebraic expressions and equations generalize relationships from specific cases.
Duration: **Credit:**	Full Year 1.0 Credit(s)
Course	*Pre-AP Algebra 2 Course framework*

Materials/Resources:	*Illustrative Mathematics Algebra II* - McGraw Hill (2020)
FPS Course Academic Expectation(s):	Exploring and Understanding Synthesizing and Evaluating
Year at a Glance (Units):	Unit 1: Modeling with Functions (~ 7-9 weeks) Unit 2: The Algebra of Functions (~ 6-8 weeks) Unit 3: Function Families (~ 9-11 weeks) Unit 4: Trigonometric Functions (~ 6-8 weeks)

Unit Number and Title:	Unit 1: Modeling with Functions
Duration:	~ 7-9 weeks
Unit Overview:	In the first unit of the course, students build upon their previous experience with linear, quadratic, and exponential functions. These important functions form the foundation upon which other functions introduced in this course are built. Unit 1 focuses on using functions to model real-world data sets and contextual scenarios. This focus on modeling provides authentic opportunities for students to investigate and confirm the defining characteristics of linear, quadratic, and exponential functions while simultaneously reinforcing procedural fluency with these function families. Throughout Algebra 2, students are expected to take ownership of the mathematics they use by crafting arguments for why one type of function is better than another for modeling a particular data set or contextual scenario. This allows students to develop a deeper understanding of these foundational functions as they drive the mathematical modeling process themselves. This requires a more thorough understanding of modeling than prior mathematics courses, in which students were asked to explain why a given function type was an appropriate model for a given data set or contextual scenario.
Learning Goals	
Standard(s):	**ALGEBRA** **Creating Equations (CED)** **Create equations that describe numbers or relationships.** A.CED.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions*. A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods

	Seeing Structure in Equations (SSE) **Interpret the structure of expressions.** A.SSE.1 Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret P(1 + r)n as the product of P and a factor not depending on P* **Write expressions in equivalent forms to solve problems.** A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression **Reason with Equations and Inequalities (REI)** **Represent and solve equations and inequalities graphically.** A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). A.REI.11 Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. **FUNCTIONS** **Building Functions (BF)** **Build a function that models a relationship between two quantities.** F.BF.1 Write a function that describes a relationship between two quantities.

	Build new functions from existing functions. F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. **Interpreting Functions (IF)** **Interpret functions that arise in applications in terms of the context.** F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*. F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.* F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. **Analyze functions using different representations.** F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

	b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. **Linear, Quadratic, and Exponential Models (LE)** **Construct and compare linear, quadratic, and exponential models and solve problems.** F.LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. **Statistics and Probability** **Interpreting Categorical and Quantitative Data (ID)** **Summarize, represent, and interpret data on two categorical and quantitative variables.** S.ID.5 ~~Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies).~~ Recognize possible associations and trends in the data. S.ID.6 Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models. b. Informally assess the fit of a function by plotting and analyzing residuals. S-ID 8 Compute (using technology) and interpret the correlation coefficient of a linear fit.

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Essential Question(s):	• Can two algebraic expressions that appear to be different be equivalent? • How do we collect, organize and display data using appropriate statistical and graphical methods? • How can concepts of transformations be applied to functions?
Enduring Understanding(s):	Students will understand that… • Many bivariate data sets can be appropriately modeled by linear, quadratic, or exponential functions because the relationships between the quantities exhibit characteristics similar to those functions. • Mathematical functions almost never perfectly fit a real-world context, but a function model can be useful for making sense of that context. • Average rate of change allows us to understand multifaceted relationships between quantities by modeling them with linear functions.
Learning Goal(s): *Students will know and will be able to use their learning to:* (Content/ Skills)	**Content:** (Students will know/understand…) • Linear, Quadratic, and Exponential functions can be used to model real world data sets. • The residual and residual plot, as well as correlation coefficient and coefficient of determination, are used to determine the appropriateness of the regression model to describe real world data. • A linear function can be expressed in slope-intercept form to reveal the constant rate of change and the initial value, or in point slope form to reveal the constant rate of change and one ordered pair that satisfies the relationship. • A quadratic function can be expressed in vertex form to reveal its maximum or minimum value; in factored form to reveal the zeros of the function, which often correspond to the boundaries of the contextual domain; or in standard form to reveal the initial • An exponential function can be expressed in the form $f(x) = a(1 + r)^x$ to reveal the percent change in the output, r, for a one-unit change in the input, or in the form $f(x) = a \bullet b^{\frac{x}{n}}$ to reveal the growth/decay factor, b, over an n-unit change in the input.

	● A function within a function family that best fits a data set minimizes the error of the function model, which is often quantified by the sum of the squares of the residuals. ● An appropriate model for a bivariate data set can be used to predict values of the dependent variable from a value of the independent variable. ● Functions that model a data set are frequently only useful over their contextual domain. ● The average rate of change is calculated and then analyzed as a function over an interval to estimate values of a function within or near the interval in question. ● Piecewise-defined functions are graphed, evaluated, and analyzed on a set of non-overlapping intervals. ● Piecewise functions can be used to model real world situations. ● The absolute value function is analyzed both algebraically and graphically. ● Solve absolute value equations to obtain one or more values of the variable in question. **Skills:** (Students will be able to…) ● Identify a function family that would appropriately model a data set or contextual scenario. ● Use residual plots to determine whether a function model appropriately models a data set. ● Construct a representation of a linear, quadratic, or exponential function both with and without technology. ● Use a function that models a data set or contextual scenario to predict values of the dependent variable. ● Interpret the average rate of change of a function over a given interval, including contextual scenarios. ● Predict values of a function using the average rate of change and an input-output pair of a function model. ● Construct a representation of a piecewise defined function. ● Evaluate a piecewise-defined function at specified values of the domain.

	● Construct a representation of an absolute value function. **In addition to the learning goals above, students enrolled in the honors level will be able to use their learning to:** ● Model real world data sets using Linear, Quadratic, and Exponential functions in addition to other types of functions, such as, but not limited to, Power, Logistic, Sinusoidal, Variable Mean functions can be used to fit other types of real world data sets ● Graph, evaluate, and analyze piecewise-defined functions on a set of non-overlapping intervals. These functions will include both linear and nonlinear equations.

Unit Number and Title:	Unit 2: The Algebra of Functions
Duration:	~ 6-8 weeks
Unit Overview:	In this unit, students develop a conceptual understanding of the algebra of functions and build procedural fluency with function notation. Students tend to think about transformations of functions and composition of functions as unrelated topics. In this unit, students connect these important concepts to develop a more coherent understanding of functions by first exploring function composition, a new operation that chains functions together in a sequence. Once students understand the power of function composition, they work to see how function transformations are a special case of composition in which a given function is composed with a linear function. The unit culminates in an exploration of inverses—the mathematical concept of undoing—through inverse operations and inverse functions. Students develop familiarity with inverse operations through their prior school experiences with addition and multiplication, and their respective inverses, subtraction and division. In this unit, the inverse operation of exponentiating—taking a logarithm—is introduced. From prior coursework, students know that a function associates each input with one output. In this course, students learn that if a function has an inverse function, it associates an output back to its input. By considering inverses as both operations and functions, students develop a deep understanding of this critical concept.
Learning Goals	
Standard(s):	**FUNCTIONS** **Building Functions (BF)** **Build new functions from existing functions.** F.BF.1 Write a function that describes a relationship between two quantities. b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*

	c. (+) Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time. F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. F.BF.4 Find inverse functions a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, $f(x) = 2x^3$ or $f(x) = \frac{x+1}{x-1}$ for $x \neq 1$. b. Verify by composition that one function is the inverse of another. c. Read values of an inverse function from a graph or a table, given that the function has an inverse. d. Produce an invertible function from a non-invertible function by restricting the domain. **Interpreting Functions (IF)** **Understand the concept of a function and use function notation.** F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then *f(x)* denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$. F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Interpret functions that arise in applications in terms of the context.

F.IF.4

For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*.

F.IF.5

Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function*.

Analyze functions using different representations.

F.IF.8

Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

F.IF.9

Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions)..

NUMBER AND QUANTITY

Quantities (Q)

Reason quantitatively and use units to solve problems.

N.Q.2

Define appropriate quantities for the purpose of descriptive modeling.

N.Q.3

Choose a level of accuracy appropriate to limitations on measurement when reporting quantities

Essential Question(s):	● What impact do different aspects of constants affect the transformation of a function? ● How do inverse functions allow one to solve different problems?
Enduring Understanding(s):	Students will understand that… ● Composing functions allows simpler functions to be combined to construct a function model that more appropriately captures the characteristics of a contextual scenario. ● Transformations are a special kind of composition. When one of the functions being composed consists only of addition or multiplication, the effects on the other function are straightforward to determine. ● An inverse function defines the way to determine the input value that corresponds to a given output value.
Learning Goal(s): *Students will know and will be able to use their learning to:* (Content/ Skills)	**Content:** (Students will know/understand…) ● Composing functions is a process in which the output of one function is used as the input of another function. Include multiple notations: $f(g(x))$ and $(f \circ g)$(x) ● Show composition through algebra, use of a table and graphs. ● Any function can be expressed as the composition of two or more functions. One of these functions can be the identity function, $f(x) = x$. ● Transformations of functions: ○ The transformation $f(x) + k$ is an additive transformation of $f(x)$. It has the effect of adding k to the output of $f(x)$. It translates the graph of $f(x)$ vertically up by k units. ○ The transformation $f(x - h)$ is an additive transformation of $f(x)$. It has the effect of adding h to the input of $f(x)$. It translates the graph of $f(x)$ horizontally right by h units. ○ The transformation $a \bullet f(x)$ is a multiplicative transformation of $f(x)$ where $a \neq 0$. It has the effect of multiplying a to the output values of $f(x)$. If a is greater than one, the graph is vertically dilated by a factor of a. If a is negative, the graph reflects over the x-axis. ○ The transformation $a \bullet f(x)$ is a multiplicative transformation of $f(x)$ where $a \neq 0$. It

	has the effect of multiplying a to the output values of $f(x)$. If a is greater than one, the graph is vertically dilated by a factor of a. If a is negative, the graph reflects over the x-axis. ● A function transformation is a sequence of additive and multiplicative transformations of $f(x)$. The order in which the transformations are applied matters. For the equation $a \cdot f(x - h) + k$, the multiplicative transformation must be applied first to the output values of a function. The additive transformations, h and k, must be applied to the input and output values respectively. ● For algebraic representations of an equation, inverse operations, such as squaring/square rooting and cubing/cube rooting, can be used to determine the input values that correspond to a specified output value. ● Determine inverse graphically, using a table and algebraically. ● For graphical representations of an equation, identifying all ordered pairs that lie on the intersection of the line $y = k$ and the graph of $y = f(x)$ provides all input values that correspond with the output value k. ● A function f has an inverse function on a specified domain if each output value of f corresponds to exactly one input value in that domain. ● A function f is called invertible on a specified domain if there exists an inverse function, f^{-1}, such that $f(a) = b$ implies $f^{-1}(b) = a$. ● There are multiple ways to restrict the domain of a function so that the function is invertible. The appropriate domain restrictions for making a function invertible may depend on the context. Specifically restricting the domain for a quadratic function. ● The graph of the inverse function of f is a reflection of the graph of $y = f(x)$ across the line $y = x$. ● The domain and range of f^{-1} are the range and domain of f, respectively. ● Using composition to prove inverses: $f(g(x)) = x$ and $g(f(x)) = x$. **Skills:** (Students will be able to…)

	• Determine the output value of the composition of two or more functions for a given input value when the functions have the same or different representations. • Construct a representation of a composite function when the functions being composed have the same or different representations. • Express a given algebraic representation of a function in an equivalent form as the composition of two or more functions. • Compare a function $f(x)$ with an additive transformation of f, that is, $f(x - h)$ or $f(x) + k$. • Compare a function $f(x)$ with a multiplicative transformation of $f(x)$, that is, $a \cdot f(x)$ or $f(ax)$. • Construct a representation of the transformation of a function. • Determine all input values that correspond to a specified output value given a function model on a specified domain. • Determine a domain over which the inverse function of a specified function is defined. • Construct a representation of the inverse function given a function that is invertible on its domain. • Verify that one function is an inverse of another function using composition. **In addition to the learning goals above, students enrolled in the honors level will be able to use their learning to:** • Compose extending to all functions with emphasis on non-linear and non-quadratic. • Transform to include horizontal dilation ○ The transformation $f(ax)$ is a multiplicative transformation of $f(x)$ where $a \neq 0$. It has the effect of multiplying $\frac{1}{a}$ to the input values of the function and is horizontally dilated by a factor of $\frac{1}{a}$. If a is negative, the graph reflects over the y-axis. • Find the inverse of an exponential equation by being introduced to a logarithm. • Find the inverse of absolute value by restricting the domain.

Unit Number and Title:	Unit 3: Function Families
Duration:	~ 9-11 Weeks
Unit Overview:	Explorations of function families are an important component of any Algebra 2 course because they expand the repertoire of functions students can draw upon to model real-world phenomena. Throughout this unit, students learn that a parent function and its transformations form a function family. All functions in the same function family share some properties with each other. Key concepts in this unit intentionally focus students' thinking on how function families are related in meaningful ways. The structure of the unit is intended to help students construct a network of connections among these function families. As with all explorations of functions throughout Algebra 2, the emphasis is on contextual scenarios that can be effectively modeled by each function family.
Learning Goals	
Standard(s):	**ALGEBRA** **Reasoning with Equations and Inequalities (REI)** **Understand solving equations as a process of reasoning and explain the reasoning.** A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. A.REI.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. **Reasoning with Equations and Inequalities (REI)** **Solve equations and inequalities in one variable.** A.REI.4 Solve quadratic equations in one variable.

	b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b. **Represent and solve equations and inequalities graphically.** A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). A.REI.11 Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. **Creating Equations (CED)** **Create equations that describe numbers or relationships.** A.CED.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions*. A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. **Seeing Structure in Expressions (SSE)** **Interpret the structure of expressions.** A.SSE.1 Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients.

	b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.* A.SSE.2 Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.* (factoring perfect cubes, etc). A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. a. Factor a quadratic expression to reveal the zeros of the function it defines. c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.* **Arithmetic with Polynomials and Rational Expressions (APR)** **Perform arithmetic operations on polynomials.** A.APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. **Understand the relationship between zeros and factors of polynomials.** A.APR.2 Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by *$x - a$* is *$p(a)$*, so *$p(a) = 0$* if and only if *$(x - a)$* is a factor of *$p(x)$*. A.APR.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Use polynomial identities to solve problems.

A.APR.4

Prove polynomial identities and use them to describe numerical relationships.

FUNCTIONS

Linear, Quadratic, and Exponential Models (LE)

Construct and compare linear, quadratic, and exponential models and solve problems.

F.LE.2

Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

F.LE.4

For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where *a, c,* and *d* are numbers and the base b is 2, 10, or *e*; evaluate the logarithm using technology.

Interpreting Functions (IF)

Understand the concept of a function and use function notation.

F.IF.2

Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Interpret functions that arise in applications in terms of the context.

F.IF.4

For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*.

F.IF.5

	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.* **Analyze functions using different representations.** F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. b. Graph square root, cube root~~, and piecewise-defined~~ functions, ~~including step functions and absolute value functions~~. c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. d. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. e. Graph exponential and logarithmic functions, showing intercepts and end behavior, ~~and trigonometric functions, showing period, midline, and amplitude~~. F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum. **Building Functions (BF)** **Build a function that models a relationship between two quantities.** F.BF.1 Write a function that describes a relationship between two quantities.

Build new functions from existing functions.

F.BF.3

Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

F.BF.5

Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

NUMBER AND QUANTITY

Complex Number System (CN)

Perform arithmetic operations with complex numbers.

N.CN.1

Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.

N.CN.2

Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

Use complex numbers in polynomial identities and equations.

N.CN.7

Solve quadratic equations with real coefficients that have complex solutions.

N.CN.8

Extend polynomial identities to the complex numbers. *For example, rewrite* $x^2 + 4 = (x + 2i)(x - 2i)$.

N.CN.9 (+)

Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

	The Real Number System (RN) **Extend the properties of exponents to rational exponents.** N.RN.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define* $5^{\frac{1}{3}}$ *to be the cube root of 5 because we want* $\left(5^{\frac{1}{3}}\right)^{3} = \left(5^{3}\right)^{\frac{1}{3}}$ *so* $\left(5^{\frac{1}{3}}\right)^{3}$ *must equal 5.* N.RN.A.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.
Essential Question(s):	● How can polynomials be simplified and applied to solve problems? ● Can two algebraic expressions that appear to be different be equivalent? ● How are the properties of real numbers related to polynomials? ● How can we extend from real numbers to complex numbers? ● How can we solve equations with complex and real roots? ● How do we apply properties of exponents to whole numbers and rational exponents?
Enduring Understanding(s):	Students will understand that… ● A function is a special mathematical relationship between two variables that can often be used to make sense of observable patterns in contextual scenarios. ● Functions in a family have similar properties, similar algebraic representations, and graphs that share key features.
Learning Goal(s): *Students will know and will be able to use their learning to:* (Content/ Skills)	**Content:** (Students will know/understand…) ● Any exponential model can be expressed in any base, including the natural base, using properties of exponents and/or function composition and properties of logarithms. The natural base e, which is approximately 2.718, is often used as the base in exponential functions that model contextual scenarios involving continuously compounded interest.

	- Evaluate logarithms of various bases. - A horizontal dilation of the graph of an exponential function is equivalent to a change of the base of the function, because $f(x) = b^{k}$ can be expressed as $f(x) = (b^{k})^{x}$, where $b^{k}$is a constant. - The graph of a logarithmic function exhibits vertical asymptotic behavior. - An algebraic representation of a logarithmic function with base b is a transformation of $f(x) = log_{b}(x)$, where $b \neq 1$ and $b > 0$. - A logarithmic function can be expressed in any base using properties of logarithms. Logarithmic functions are often expressed in base e, which is called the natural logarithm and is notated as "ln" rather than log_{e}. - A horizontal dilation is equivalent to a vertical translation because $f(x) = log_{b}(kx)$ can be rewritten as $f(x) = log_{b}(k) + log_{b}(x)$, where $log_{b}(k)$ is a constant. - Raising the input of a logarithmic function to a power of k results in a vertical dilation of the graph because $f(x) = log_{b}(x^{k})$ can be rewritten as $f(x) = k * log_{b}(x)$. - The inverse of the exponential function $f(x) = b^{x}$ is the logarithmic function $g(x) = log_{b}(x)$ and vice versa. - Equations involving exponential functions can be solved algebraically by taking a logarithm or have solutions that can be estimated by examining a graph of the function. - Equations involving logarithmic functions can be solved algebraically by exponentiating or have solutions that can be estimated by examining a graph of the function. - Factoring of quadratic and high order polynomials. - A polynomial function is a function whose algebraic representation can be expressed as $f(x) = a_{n}x^{n} + a_{n-1}x^{n-1} +...+ a_{1}x + a_{0}$, where $a_{n} \neq 0$.

	• Scenarios involving areas of figures and surface areas and volumes of solids and maximum or minimum values are often well modeled by polynomial functions. • The standard form of a polynomial function reveals the degree of the polynomial, *n*, which is the highest power of all the terms. • A linear factor, $(x - a)$, of a polynomial function, *p*, corresponds to a zero (or root) of p at $x = a$ because $p(a) = 0$. • A polynomial function factored into a product of linear factors reveals the x-intercepts of the graph of the function, which are the real zeros of the polynomial. The total number of real zeros is at most equal to the degree of the polynomial function. • A local maximum or minimum of a nonconstant polynomial function corresponds to the output value of the point at which the function switches between increasing to decreasing in either order. • Between every two real zeros of a nonconstant polynomial function, there must be at least one input value corresponding to a local maximum or minimum. • The end behavior of a polynomial function can be determined visually from its graph or by examining the degree of the polynomial and the sign of its leading coefficient. • If a linear factor $(x - a)$ of a polynomial function has an even power, then the signs of the output values are the same for input values near $x = a$. For these polynomials, the graph will be tangent to the x-axis at $x = a$. • Every complex number has the form $a + bi$ where *a* and *b* are real numbers and $i^2 = -1$.The real part of the complex number is *a* and the imaginary part of the complex number is *b*. • Adding or subtracting two complex numbers involves performing the indicated operation with the real parts and the imaginary parts separately. Multiplying two complex numbers is accomplished by applying the distributive property and using the relationship $i^2 = -1$. • Complex numbers occur naturally as solutions to quadratic equations with real coefficients. Therefore, verifying that a complex number is a solution of a quadratic equation requires adding

	and multiplying complex numbers. ● Quantities that are inversely proportional are often well modeled by rational functions. ● Rational functions often have restricted domains. These restrictions correspond to the zeros of the polynomial in the denominator and often manifest in the graph as vertical asymptotes. ● Zeros of a rational function correspond to the zeros/x-intercepts of the polynomial in the numerator that are in the domain of the function. The x-intercepts of the graph of a rational function correspond to the zeros of the function. ● The end behavior of a rational function can be determined by examining the behavior of a function formed by the ratio of the leading term of the numerator to the leading term of the denominator. ● The square root function, $f(x) = \sqrt{x}$ is the inverse of the quadratic function $g(x) = x^2$ over the restricted domain $[0, \infty)$. Therefore, the graph of $y = \sqrt{x}$ resembles the graph of $y = x^2$ for $x \geq 0$ reflected across the line $y = x$. ● The domain of a square root function, a function transformation of $f(x) = \sqrt{x}$, corresponds to the set of input values for which the expression under the radical is nonnegative. ● Real-world scenarios involving distance traveled and elapsed time for free-falling objects are well modeled by square root functions. ● The cube root function, $f(x) = \sqrt[3]{x}$ is the inverse of the cubic function $g(x) = x^3$. Thus, the graph of $y = \sqrt[3]{x}$ resembles the graph of $y = x^3$ reflected across the line $y = x$. ● The domain of a cube root function, a function transformation of $f(x) = \sqrt[3]{x}$, is all real numbers. ● Real-world scenarios involving side lengths of solids with a known volume are well modeled by cube root functions. ● The inverse of a quadratic function is a square root function. ● The algebraic representation of the inverse of a quadratic function $y = f(x)$ is determined by first expressing f in vertex form and then using inverse operations to express x in terms of y.

	- Since many output values of quadratic functions are each associated with multiple input values, constructing an inverse of a quadratic function requires restricting the domain of the function so the function is invertible. For values of x in the restricted domain of the quadratic function f, $f^{-1}(f(x)) = x$. - Equations involving square roots and cube roots arising from contextual scenarios can be solved algebraically using inverse operations, such as squaring or cubing, or have solutions that can be estimated by examining an associated graph of the function models. - Solving equations by squaring can introduce values called extraneous solutions, which are not actual solutions of the equation. **Skills:** (Students will be able to…) - Construct a representation (graph, table and equation) of exponential, logarithmic, polynomial, rational, square root, and cube root functions. - Construct a representation of an exponential function using the natural base, e. - Identify key features of these graphs. - Express an exponential, logarithmic, and polynomial function in an equivalent form to reveal properties of the graph and/or the contextual scenario. - Construct a representation of the inverse function of an exponential, logarithmic and quadratic function. - Solve equations involving exponential or logarithmic functions, and square root or cube root functions, including those arising from contextual scenarios. - Perform arithmetic with complex numbers. **In addition to the learning goals above, students enrolled in the honors level will be able to use their learning to:** - Understand that horizontally translating the graph of an exponential function can also be thought of as a vertical dilation of the graph because $f(x) = b^x * b^k$, where b^k is a constant.

	• Factor a binomial out of polynomials. • Use division to determine oblique asymptotes. • Data sets and/or contextual scenarios that exhibit a roughly constant ratio of the inputs for equal additive changes in the outputs are appropriately modeled by logarithmic functions. • Factor perfect cubes. • Use the binomial theorem to expand $(a+b)^n$.

Unit Number and Title:	Unit 4: Trigonometric Functions
Duration:	~ 6-8 weeks
Unit Overview:	This unit provides an exploration of trigonometric functions. Trigonometry is the branch of mathematics that connects two fundamental geometric objects: triangles and circles. In Geometry, students learned that the trigonometric ratios relate acute angle measures to ratios of side lengths in right triangles. Algebra 2 extends those relationships to include all real numbers. When the domains of trigonometric functions include angle measures greater than 90°, including greater than 360°, these functions are far more useful in modeling contextual scenarios that involve periodic phenomena, such as the rotation of objects, the height of a Ferris wheel car, or the ebb and flow of tides. Beginning with the introduction of radians as units of angle measure, the unit continues with an investigation of the sine and cosine functions and their transformations, collectively referred to as sinusoidal functions. Finally, students use inverse trigonometric functions to solve problems related to circular and periodic motion.
Learning Goals	
Standard(s):	**FUNCTIONS** **Trigonometric Functions (TF)** **Extend the domain of trigonometric functions using the unit circle.** F.TF.1 Understand the radian measure of an angle as the length of the arc on the unit circle subtended by the angle. F.TF.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. F.TF.3 (+)

	Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosine, and tangent for x, π + x, and 2π – x in terms of their values for x, where x is any real number. F.TF.4 (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. **Model periodic phenomena with trigonometric functions.** F.TF.B.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. **Prove and apply trigonometric identities.** F.TF.8 Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle. **Interpreting Functions (IF)** **Understand the concept of a function and use function notation.** F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$. F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. **Interpret functions that arise in applications in terms of the context.** F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features

	given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*. **Analyze functions using different representations.** F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. e. Graph ~~exponential and logarithmic functions, showing intercepts and end behavior, and~~ trigonometric functions, showing period, midline, and amplitude. **Building Functions (BF)** **Build new functions from existing functions.** F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
Essential Question(s):	● How do we extend known rules of functions to trigonometric functions (transformations and operations)? ● How do we graph by hand and using technology? ● How do we model periodic data to solve problems?
Enduring Understanding(s):	Students will understand that… ● Trigonometry connects the study of circles and the study of right triangles. ● Real-world contexts that exhibit periodic behavior or circular motion can be modeled by trigonometric functions.
Learning Goal(s): *Students will know and*	**Content:** (Students will know/understand…) ● A radian represents an arc length that is in direct proportion to its central angle.

will be able to use their learning to: (Content/ Skills)	● The relationship between angles measured in degrees and radians can be described by the following proportion: $\frac{measure\ in\ degrees}{measure\ in\ radians} = \frac{180}{\pi}$. ● A unit circle has a radius of 1 unit of measure, based on the context of a scenario. ● The Pythagorean theorem for trigonometric functions, $sin^2(\theta) + cos^2(\theta) = 1$, can be deduced from the fact that a circle of radius r centered at the origin is the solution set to the equation $x^2 + y^2 = r^2$ and that the coordinates of a point on that circle are given by $x = rcos(\theta)$ and $y = rsin(\theta)$. ● The coordinates of the point at which the terminal ray of an angle in standard position intersects a unit circle are determined by $(x, y) = (cos(\theta),\ sin(\theta))$. The tangent of the angle's measure is the slope of the terminal ray. Thus, $tan(\theta) = \frac{sin(\theta)}{cos(\theta)}$. ● In the context of circular motion, the function $f(\theta) = sin(\theta)$ relates the measure of an angle in standard position to the vertical displacement from the origin of a point on the unit circle and the function $f(\theta) = cos(\theta)$ relates the measure of an angle in standard position to the horizontal displacement from the origin of a point on the unit circle. ● Sinusoidal functions include the functions $f(x) = sin(x)$ and $f(x) = cos(x)$ and their transformations. Transformations include amplitude change, period change and vertical and horizontal transformations. ● Model real-world periodic behavior or circular motion using sine or cosine functions. **Skills:** (Students will be able to…) ● Use the radian measure of an angle to relate the radius of a circle to the length of the arc subtended by that angle. ● Determine when two angles in the coordinate plane are coterminal. ● Construct a representation of a sinusoidal function. ● Determine the exact coordinates of any point on a circle centered at the origin. ● Identify key characteristics of a sine and cosine function.

	- Construct a sine or cosine function to model a periodic phenomenon that has a specified frequency, period, amplitude, and phase shift. - Solve equations involving trigonometric functions. **In addition to the learning goals above, students enrolled in the honors level will be able to use their learning to:** - Construct a representation of a tangent function. - Construct a representation of an inverse trigonometric function and understand restrictions on the domain. - Contextual scenarios involving the height of a rising or falling object or slopes of lines are often modeled with a tangent function. - Secant, cosecant, and cotangent are the names given to trigonometric functions formed by quotients of sinusoidal functions, defined as $sec(\theta) = \frac{1}{cos(\theta)}$, $csc(\theta) = \frac{1}{sin(\theta)}$, $cot(\theta) = \frac{cos(\theta)}{sin(\theta)}$. - The inputs and outputs of inverse trigonometric functions are switched from their corresponding trigonometric functions, so the output of an inverse trigonometric function is often interpreted as an angle measure and the input is a value in the range of the corresponding trigonometric function. - The inverse trigonometric functions arcsine, arccosine, and arctangent (also represented as sin^{-1}, cos^{-1}, tan^{-1}) have restricted domains. - Identify key characteristics and values of functions that are defined by quotients of sinusoidal functions.